Spectral Efficiency Evaluation for Selection Combining Diversity Schemes under Worst Case of Fading Scenario

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Abstract: The results of spectral efficiencies for optimum rate adaptation with constant transmit power (ORA) and channel inversion with fixed rate (CIFR) schemes over uncorrelated diversity branch with Selection Combining (SC) available so far in literature are not applicable for Nakagami-0.5 fading channels. This paper derived closed-form expressions for the spectral efficiency of dual-branch SC over uncorrelated Nakagami-0.5 fading channels. This spectral efficiency is evaluated under ORA and CIFR schemes. Since, the spectral efficiency expression under ORA scheme contains an infinite series, hence bounds on the errors resulting from truncating the infinite series have been derived The corresponding expressions for Nakagami-0.5 fading are called expressions under worst fading condition with severe fading. Finally, numerical results are presented, which are then compared to the spectral efficiency results which have been previously published for ORA and CIFR schemes. It has been observed that by employing SC, spectral efficiency improves under ORA, but does not improve under CIFR.

Keywords: Dual-branch, Channel inversion with fixed rate, Nakagami-0.5, Optimum rate adaptation with constant transmit power, Selection combining, Spectral efficiency.

1. Introduction

Wireless communication services, such as wireless personal area networks, satellite-terrestrial services, wireless mobile communication services, wireless local-area networks, and internet access have been growing at a rapid pace in recent years. These services require high data rate. Thus, channel capacity is of fundamental importance in the design of wireless communication systems as it determines the maximum achievable data rate of the system. Since wireless mobile channels are subjected to fading, which degrades the data rate performance. The channel capacity in fading environment can be improved by employing diversity combining and / or adaptive transmission schemes [1]-[5].

Diversity combining is known to be a powerful technique that can be used to combat fading in wireless mobile environment. Maximal ratio combining, equal gain combining and SC are most prevalent diversity combing techniques [3]-[4].

Adaptive transmission is another effective scheme that can be used to overcome fading. Adaptive transmission requires accurate channel estimation at the receiver and a reliable feedback path between the estimator and the transmitter [6]. There are four adaptation transmission schemes such as ORA, CIFR, optimum power and rate adaptation (OPRA) and truncated channel inversion with fixed Rate (TIFR) [6]-[8]. Numerous researchers have worked on the study of channel capacity over different fading channels. We discuss here some representative examples. Specifically, [3]-[4] discuss the channel capacity over correlated Nakagami-m ($m \geq 1$ & $m < 1$) fading channels under ORA and CIFR schemes with different diversity combining techniques. In [7], the channel capacity over uncorrelated Nakagami-m ($m \geq 1$) fading channels with MRC and without diversity under different adaptive transmissions schemes was analyzed. Expressions for the capacity over uncorrelated Rayleigh fading channels with MRC and SC under different adaptive transmission schemes were obtained in [8]. An analytical performance study of the channel capacity for correlated generalized gamma fading channels with dual-branch SC under the different power and rate adaptation schemes was introduced in [9]. The channel capacity of Nakagami-m ($m \geq 1$) fading channel without diversity was derived in [10] for different adaptive transmission schemes. In [11], channel capacity of dual-branch SC and MRC systems over correlated Hoyt fading channels using different adaptive transmission schemes was presented. In [12], expression for the ergodic capacity of MRC over arbitrarily correlated Rician fading channels was derived. In [13], an expression for lower and upper bounds in the channel capacity expression for uncorrelated Rician and Hoyt fading channels with MRC using ORA scheme were obtained. The analytical study of the capacity under $k-\mu$ fading and Weibull fading channels with OPRA, ORA, CIFR and TIFR adaptation transmission schemes using different diversity systems was presented in [14]. In [15], an analytical performance study of the channel capacity for uncorrelated Nakagami-0.5 with dual-branch MRC using OPRA and TIFR was obtained. In [16], the channel capacity over correlated Nakagami-0.5 fading channels under OPRA and TIFR schemes with MRC was discussed. An analytical performance study of the channel capacity for uncorrelated Nakagami-0.5 fading channels with dual-branch SC under OPRA, and TIFR was introduced in [17]. The Nakagami-0.5 model has been widely used in general to study wireless mobile communication system performance. Results obtained for Nakagami-0.5 will have great practical usefulness, they will be of theoretical interest as a worst fading case. This paper fills this gap by presenting an analytical performance study of the channel capacity of dual-branch SC over uncorrelated Nakagami-0.5 fading channels using ORA, and CIFR schemes.

In this paper, SC has been considered which is one of the least complex diversity combining techniques [18].

The remainder of this paper is organized as follows: In Section 2, the channel model is defined. In Section 3, spectral efficiency of no diversity and dual-branch SC over Nakagami-0.5 fading channels are derived for ORA and CIFR schemes. In Section 4, several numerical results are presented and analyzed, whereas in Section 5, concluding remarks are given.

2. Channel Model

We assume slowly-varying Nakagami-m flat fading channel. The probability distribution function (pdf) of instantaneous received SNR (γ) of this fading channel is gamma distributed given by [7]

$$p_\gamma(\gamma)=\frac{\gamma^{m-1}}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^m \exp\left(\frac{-m\gamma}{\bar{\gamma}}\right),\quad \gamma\geq 0\ \&\ m\geq 0.5 \tag{1}$$

where m is the Nakagami fading parameter, which measures the amount of fading, $\bar{\gamma}$ is the average received SNR, and $\Gamma(.)$ is the gamma function. For different values of m, this expression simplifies to several important distributions describing fading models. Like $m=0.5$ corresponds to the highest amount of fading, $m=1$ corresponds to Rayleigh distribution, $m\geq 1$ corresponds to Rician distribution, and as $m\to\infty$, the distribution converges to a nonfading AWGN from [19].

In case of no diversity the pdf under worst case of fading using (1) is

$$p_\gamma(\gamma)=\frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{2\pi\bar{\gamma}\gamma}},\quad \gamma\geq 0 \tag{2}$$

Assuming independent branch signals and equal average received SNR, the pdf of the received SNR at the output of dual-branch SC under Nakagami-m fading channels is given by [19]-[20] is

$$p_\gamma(\gamma)=\frac{2\gamma^{m-1}}{\Gamma(m)}\left(\frac{m}{\bar{\gamma}}\right)^m \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)\left[1-Q_m\left(0,\sqrt{\frac{2m\gamma}{\bar{\gamma}}}\right)\right],\quad \gamma\geq 0 \tag{3}$$

where $\bar{\gamma}$ is the average received SNR, $m\,(m\geq 0.5)$ is the fading parameter, and $Q_m(.)$ is the Marcum Q-function, which can be represented, when m is not an integer, as given in [19]

$$Q_m\left(0,\sqrt{\frac{2m\gamma}{\bar{\gamma}}}\right)=\frac{\Gamma\left(m,\frac{m\gamma}{\bar{\gamma}}\right)}{\Gamma(m)}$$

where $\Gamma[.,.]$ is the complementary incomplete gamma function.

As we consider worst case of fading, then by [21]

$$Q_{0.5}\left(0,\sqrt{\frac{2\times 0.5\gamma}{\bar{\gamma}}}\right)=\frac{\Gamma\left(0.5,\frac{0.5\gamma}{\bar{\gamma}}\right)}{\Gamma(0.5)}=erfc\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right)$$

where $erfc(.)$ is called complementary error function. So that

$$1-erfc\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right)=erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right)$$

Hence, the pdf of dual-branch SC under worst case of fading using above mathematical transformation as given in [17]

$$p_\gamma(\gamma)=\sqrt{\frac{2}{\pi\gamma\bar{\gamma}}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right),\quad \gamma\geq 0 \tag{4}$$

3. Spectral Efficiency

In this section, we present closed-form expressions for the spectral efficiency of uncorrelated Nakagami-0.5 fading channels with dual-branch SC and no diversity under ORA, and CIFR schemes. It is assumed that, for the above considered adaptation scheme, there exist perfect channel estimation and an error-free delayless feedback path, similar to the assumption made in [8].

3.1 ORA

The average channel capacity of fading channel with received SNR distribution $p_\gamma(\gamma)$ under ORA scheme (C_{ORA} [bit/sec]) is defined in [6] as

$$C_{ORA}=B\int_0^\infty \log_2(1+\gamma)\,p_\gamma(\gamma)\,d\gamma \tag{5}$$

where B [Hz] is the channel bandwidth.

In fact, (5) represents the capacity of the fading channel without transmitter feedback (i.e., with the channel fade level known at the receiver only).

3.1.1 Spectral efficiency in case of no diversity

Substituting (2) into (5), the average channel capacity becomes

$$C_{ORA}=B\int_0^\infty \log_2(1+\gamma)\frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{2\pi\bar{\gamma}\gamma}}d\gamma$$

$$C_{ORA}=\frac{1.443B}{\sqrt{2\pi\bar{\gamma}}}\int_0^\infty \log(1+\gamma)\frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{\gamma}}d\gamma$$

The integral can be solved using partial integration as follows

$$\int_0^\infty u\,dv=\lim_{\gamma\to\infty}(u\,v)-\lim_{\gamma\to 0}(u\,v)-\int_0^\infty v\,du$$

Let $u=\log(1+\gamma)$

then $du=\frac{d\gamma}{1+\gamma}$

Now, let $dv = \dfrac{\exp\left(\dfrac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{\gamma}} d\gamma$

After performing integral using [21], we obtain

$$v = \sqrt{2\pi\bar{\gamma}}\, erf\left[\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right]$$

Evaluating the above integral by using partial integration and after some mathematical transformation using [21]-[22], we obtain

$$C_{ORA} = \frac{1.443B}{\bar{\gamma}}\begin{bmatrix} -{}_2F_2\left(1,1,;\frac{3}{2},2;\frac{1}{2\bar{\gamma}}\right) + \bar{\gamma}\pi \times erf\,i\left(\frac{1}{\sqrt{2\bar{\gamma}}}\right) \\ -\bar{\gamma}e + \bar{\gamma}\log\left(\frac{\bar{\gamma}}{2}\right) \end{bmatrix}$$

where ${}_2F_2(.,.;.,.;.)$ is the generalized hypergeometric function and $erf\,i(.)$ is the imaginary error function.

Using that result, we obtain average channel capacity per unit bandwidth i.e. $\frac{C_{ORA}}{B}$ [bit/sec/Hz] said to be spectral efficiency as

$$\eta_{ORA} = \frac{C_{ORA}}{B} = \frac{1.443}{\bar{\gamma}}\begin{bmatrix} -{}_2F_2\left(1,1,;\frac{3}{2},2;\frac{1}{2\bar{\gamma}}\right) + \bar{\gamma}\pi \times \\ erf\,i\left(\frac{1}{\sqrt{2\bar{\gamma}}}\right) - \bar{\gamma}e + \bar{\gamma}\log\left(\frac{\bar{\gamma}}{2}\right) \end{bmatrix} \tag{6}$$

where e is the Euler-Mascheroni constant having the value approximately equal to 0.577215665 given in [19].

3.1.2 Spectral efficiency in case of dual-branch SC

Substituting (4) into (5), the average channel capacity of dual-branch SC over uncorrelated Nakagami-0.5 fading channels is

$$C_{ORA} = B\int_0^{\infty} \log_2(1+\gamma)\sqrt{\frac{2}{\pi\bar{\gamma}\gamma}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right) erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right) d\gamma \tag{7}$$

As we know that error function can be represented by [21] as

$$erf(z) = \frac{2}{\sqrt{\pi}}\sum_{n=0}^{\infty}\frac{(-1)^n z^{2n+1}}{n!(2n+1)} \tag{8}$$

Substituting (8) in (7), C_{ORA} after some mathematical transformation

$$C_{ORA} = 1.837B\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(2n+1)(2\bar{\gamma})^{n+1}} \times \int_0^{\infty}\log(1+\gamma)\,\gamma^n \exp\left[-\frac{0.5\gamma}{\bar{\gamma}}\right] d\gamma$$

Using that result we obtain spectral efficiency i.e. $\frac{C_{ORA}}{B}$ [bit/sec/Hz] as

$$\eta_{ORA} = 1.837B\sum_{n=0}^{\infty}\frac{(-1)^n}{n!(2n+1)(2\bar{\gamma})^{n+1}} \times \int_0^{\infty}\log(1+\gamma)\,\gamma^n \exp\left[-\frac{0.5\gamma}{\bar{\gamma}}\right] d\gamma \tag{9}$$

The integral can be solved using partial integration as follows

$$\int_0^{\infty} u\,dv = \lim_{\gamma\to\infty}(uv) - \lim_{\gamma\to 0}(uv) - \int_0^{\infty} v\,du$$

Let $u = \log(1+\gamma)$

then $du = \dfrac{d\gamma}{1+\gamma}$

Now let $dv = \exp\left(-\dfrac{0.5\gamma}{\bar{\gamma}}\right)\gamma^n\, d\gamma$

After performing integral using [21], we obtain

$$v = -\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)\sum_{k=0}^{n}\frac{n!}{(n-k)!}(2\bar{\gamma})^{k+1}\gamma^{n-k}$$

Evaluating integral by using partial integral and some mathematical transformation using [21]-[22], we obtain

$$\eta_{ORA} = 1.837\sum_{n=0}^{\infty}\sum_{z=0}^{n}\frac{(-1)^n \exp\left(\frac{0.5}{\bar{\gamma}}\right)\left(\frac{0.5}{\bar{\gamma}}\right)^z \Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)}{(2n+1)} \tag{10}$$

The computation of the spectral efficiency according to (10) requires the computation of an infinite series. To efficiently compute the series, we truncate the series, and present bounds for the spectral efficiency.

The spectral efficiency in (10) can be written as $\eta_{ORA} = \eta_{ORA,\,N} + \eta_{ORA,E}$, where $\eta_{ORA,\,N}$ is the expression in (10) with the infinite series truncated at the N th term as

$$\eta_{ORA,\,N} = 1.837\sum_{n=0}^{N}\sum_{z=0}^{n}\frac{(-1)^n \exp\left(\frac{0.5}{\bar{\gamma}}\right)\left(\frac{0.5}{\bar{\gamma}}\right)^z \Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)}{(2n+1)}$$

and $\eta_{ORA,E}$ is the truncation error resulting from truncating the infinite series in (10) at $n = N$.

The lower bound for η_{ORA} is derived as

$$\eta_{ORA} > \eta_{ORA,\,N} + \eta_{ORA,E-low}$$

where $\eta_{ORA,E-low}$, which is the lower bound of $\eta_{ORA,E}$

The lower bound for the spectral efficiency can be derived by using the relationship between the area of the pdf and the expression of the spectral efficiency as discuss in [13].

As we know that area of pdf $p_\gamma(\gamma)$ is equal to unity.

$$P = \int_0^{\infty} p_\gamma(\gamma)\,d\gamma = 1 \tag{11}$$

Substituting (4) into (11), we get

$$P = \int_0^{\infty}\sqrt{\frac{2}{\pi\bar{\gamma}\gamma}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right) erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right) d\gamma = 1$$

After integrating and using manipulation we get

$$P=\frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)}=1$$

$$P_{N-1}=\frac{4}{\pi}\sum_{n=0}^{N-1}\frac{(-1)^n}{(2n+1)} \tag{12}$$

And let

$$\Delta P_{N-1}=\frac{4}{\pi}\times\frac{(-1)^N}{(2N+1)} \tag{13}$$

Then

$$P_N=P_{N-1}+\Delta P_{N-1}$$

Similarly, from (10) let

$$\eta_{ORA,N-1}=1.837\sum_{n=0}^{N-1}\sum_{z=0}^{n}\frac{(-1)^n\exp\left(\frac{0.5}{\bar{\gamma}}\right)\left(\frac{0.5}{\bar{\gamma}}\right)^z\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)}{(2n+1)} \tag{14}$$

And

$$\Delta\eta_{ORA,N-1}=1.837\frac{(-1)^N\exp\left(\frac{0.5}{\bar{\gamma}}\right)}{(2N+1)}\sum_{z=0}^{N}\left(\frac{0.5}{\bar{\gamma}}\right)^z\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right) \tag{15}$$

Dividing (15) by (13), yields

$$\frac{\Delta\eta_{ORA,N-1}}{\Delta P_{N-1}}=1.443\exp\left(\frac{0.5}{\bar{\gamma}}\right)\sum_{z=0}^{N}\left(\frac{0.5}{\bar{\gamma}}\right)^z\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right) \tag{16}$$

Observing that $\left(\frac{\Delta\eta_{ORA,N-1}}{\Delta P_{N-1}}\right)$ monotonically increases with increasing N, i.e.

$$\frac{\Delta\eta_{ORA,i}}{\Delta P_i}>\frac{\Delta\eta_{ORA,N-1}}{\Delta P_{N-1}}\ \text{ for } i\geq N$$

$$\sum_{i=N}^{\infty}\Delta\eta_{ORA,i}>\frac{\Delta\eta_{ORA,N-1}}{\Delta P_{N-1}}\sum_{i=N}^{\infty}\Delta P_i=\frac{\Delta\eta_{ORA,N-1}}{\Delta P_{N-1}}\left(1-P_N\right) \tag{17}$$

Hence, the spectral efficiency in (10) can be lower bounded $\eta_{ORA,E-low}$ by using (16) and (17) as

$$\eta_{ORA}>\eta_{ORA,N}+1.443\exp\left(\frac{0.5}{\bar{\gamma}}\right)\sum_{z=0}^{N}\left(\frac{0.5}{\bar{\gamma}}\right)^z\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)\left(1-\frac{4}{\pi}\sum_{n=0}^{N}\frac{(-1)^n}{(2n+1)}\right) \tag{18}$$

The upper bound for η_{ORA} is derived as

$$\eta_{ORA}<\eta_{ORA,N}+\eta_{ORA,E-up}$$

where $\eta_{ORA,E-up}$, which is the upper bound of $\eta_{ORA,E}$

The expression in (9) can be written as

$$\eta_{ORA,E}=\frac{1.837}{2\bar{\gamma}}\sum_{n=N+1}^{\infty}\frac{1}{(2n+1)}\times\int_0^{\infty}\log(1+\gamma)\exp\left[-\frac{0.5\gamma}{\bar{\gamma}}\right]\frac{\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)^n}{n!}d\gamma \tag{19}$$

Let $a_n=\frac{1}{2n+1}$,

Then $\frac{a_{n+1}}{a_n}=\frac{2n+1}{2n+3}<1$

i.e. a_n monotonically decreases with increase of n, therefore, $\eta_{ORA,E}$ can be upper bounded as

$$\eta_{ORA,E}<\frac{1.837}{2\bar{\gamma}(2N+3)}\times\int_0^{\infty}\log(1+\gamma)\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)\sum_{n=N+1}^{\infty}\frac{\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)^n}{n!}d\gamma$$

$$\eta_{ORA,E}<\frac{1.837}{2\bar{\gamma}(2N+3)}\times\int_0^{\infty}\log(1+\gamma)\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)\left\{\sum_{n=0}^{\infty}\frac{\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)^n}{n!}-\sum_{n=0}^{N}\frac{\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)^n}{n!}\right\}d\gamma \tag{20}$$

After evaluating the integral (20) and some mathematical manipulations using [20]-[21], we obtain the upper bound $\eta_{ORA,E-up}$ for, $\eta_{ORA,E}$ as

$$\eta_{ORA,E}<\frac{1.837}{2N+3}\left[\begin{array}{l}0.5\exp\left(\frac{1}{\bar{\gamma}}\right)E_1\left(\frac{1}{\bar{\gamma}}\right)-\\ \sum_{n=0}^{N}\sum_{z=0}^{n}(-1)^n\left(\frac{0.5}{\bar{\gamma}}\right)^z\exp\left(\frac{0.5}{\bar{\gamma}}\right)\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)\end{array}\right] \tag{21}$$

Therefore, the spectral efficiency in (10) can be upper bounded as

$$\eta_{ORA}=\eta_{ORA,N}+\eta_{ORA,E}<\eta_{ORA,N}+\frac{1.837}{2N+3}\times\left[\begin{array}{l}0.5\exp\left(\frac{1}{\bar{\gamma}}\right)E_1\left(\frac{1}{\bar{\gamma}}\right)-\\ \sum_{n=0}^{N}\sum_{z=0}^{n}(-1)^n\left(\frac{0.5}{\bar{\gamma}}\right)^z\exp\left(\frac{0.5}{\bar{\gamma}}\right)\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)\end{array}\right] \tag{22}$$

where $E_1(.)$ is the exponential integral of first order.

Hence, the spectral efficiency is bounded using (18) and (22) as

$$\eta_{ORA,N} + \frac{1.837}{2N+3}\left[\begin{array}{l} 0.5\exp\left(\frac{1}{\bar{\gamma}}\right)E_1\left(\frac{1}{\bar{\gamma}}\right) - \\ \sum_{n=0}^{N}\sum_{z=0}^{n}(-1)^n\left(\frac{0.5}{\bar{\gamma}}\right)^z \exp\left(\frac{0.5}{\bar{\gamma}}\right)\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)\end{array}\right]$$

$$> \eta_{ORA} >$$

$$\eta_{ORA,N} + 1.443\exp\left(\frac{0.5}{\bar{\gamma}}\right)\sum_{z=0}^{N}\left(\frac{0.5}{\bar{\gamma}}\right)^z\Gamma\left(-z,\frac{0.5}{\bar{\gamma}}\right)\times \left(1-\frac{4}{\pi}\sum_{n=0}^{N}\frac{(-1)^n}{(2n+1)}\right) \quad (23)$$

3.2 CIFR

The average channel capacity of fading channel with received SNR distribution $p_\gamma(\gamma)$ under CIFR scheme (C_{CIFR} [bit/sec]) is defined in [6] as

$$C_{CIFR} = B\log_2\left(1+\frac{1}{\int_0^\infty\left(\frac{p_\gamma(\gamma)}{\gamma}\right)d\gamma}\right) \quad ,\gamma \geq 0 \qquad (24)$$

Channel inversion with fixed rate is the least complex technique to implement, assuming good channel estimates are available at the transmitter and receiver.

3.2.1 Spectral Efficiency in case of no Diversity

The pdf for Nakagami-0.5 fading channel is given in (2) as

$$p_\gamma(\gamma) = \frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{2\pi\bar{\gamma}\gamma}}, \quad \gamma \geq 0$$

Hence,

$$\frac{p_\gamma(\gamma)}{\gamma} = \frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{2\pi\bar{\gamma}\gamma}}\times\frac{1}{\gamma}, \quad \gamma \geq 0 \qquad (25)$$

Integrating (25) over an interval as shown below

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \int_0^\infty \frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{2\pi\bar{\gamma}\gamma}}\times\frac{1}{\gamma}d\gamma$$

Evaluating integral by some manipulation using [21], we obtain

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \frac{1}{\sqrt{2\pi\bar{\gamma}}}\left.\left(-\sqrt{\frac{2\pi}{\bar{\gamma}}}erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right) - \frac{2\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{\gamma}}\right)\right|_0^\infty$$

As we know that

$erf(0) = 0$, $erf(\infty) = 1$,

And

$$\lim_{\gamma\to\infty}\frac{\exp\left(\frac{-0.5\gamma}{\bar{\gamma}}\right)}{\sqrt{\gamma}} = 0 \text{ , as } \bar{\gamma} > 0$$

Hence

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \frac{1}{\sqrt{2\pi\bar{\gamma}}}\left[\left(-\sqrt{\frac{2\pi}{\bar{\gamma}}}(0-1) - 2(0-\infty)\right)\right]$$

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \infty$$

Putting this value of integral in (24), we get

$$C_{CIFR} = B\log_2(1) = 0 \qquad (26)$$

So, the spectral efficiency with no diversity under Nakagami-0.5 fading channel is zero.

3.2.2 Spectral Efficiency in case of dual-branch SC

The pdf of dual-branch SC over uncorrelated Nakagami-0.5 fading channel is given in (4) is

$$p_\gamma(\gamma) = \sqrt{\frac{2}{\pi\bar{\gamma}\gamma}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right), \qquad \gamma \geq 0$$

Hence

$$\frac{p_\gamma(\gamma)}{\gamma} = \sqrt{\frac{2}{\pi\bar{\gamma}\gamma}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right)\times\frac{1}{\gamma}, \qquad \gamma \geq 0 \quad (27)$$

Integrating the (27) over an interval as shown below

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \int_0^\infty \sqrt{\frac{2}{\pi\bar{\gamma}\gamma}}\exp\left(-\frac{0.5\gamma}{\bar{\gamma}}\right)erf\left(\sqrt{\frac{0.5\gamma}{\bar{\gamma}}}\right)\times\frac{1}{\gamma}d\gamma$$

After evaluating the integral using [21]-[22], we obtain

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \sum_{n=0}^{\infty}\frac{4(-1)^n}{\pi(2\bar{\gamma})^{n+1}\,n!(2n+1)}\left[-(2\bar{\gamma})^n\,\Gamma\left(n,\frac{0.5\gamma}{\bar{\gamma}}\right)\right]_0^\infty$$

As we know that

$$\Gamma(n,0) = \Gamma(n)$$

And

$$\lim_{x\to\infty}\Gamma(n,x) = 0$$

$$\int_0^\infty \frac{p_\gamma(\gamma)}{\gamma}d\gamma = \sum_{n=0}^{\infty}\frac{4(-1)^n}{\pi(2\bar{\gamma})n!(2n+1)}\times\Gamma(n) = \infty$$

Putting this value of integral in (24), we get

$$C_{CIFR} = B\log_2(1) = 0 \qquad (28)$$

4. Numerical Results and Analysis

In this section, various performance evaluation results for the spectral efficiency have been obtained using dual-branch SC and no diversity under worst fading condition. These results also focus on spectral efficiency comparisons between the different adaptive transmission schemes.

In Fig. 1, the spectral efficiency under ORA scheme is plotted as a function of the average received SNR per branch $\bar{\gamma}$. As expected, by increasing $\bar{\gamma}$ and/or employing diversity, spectral efficiency improves.

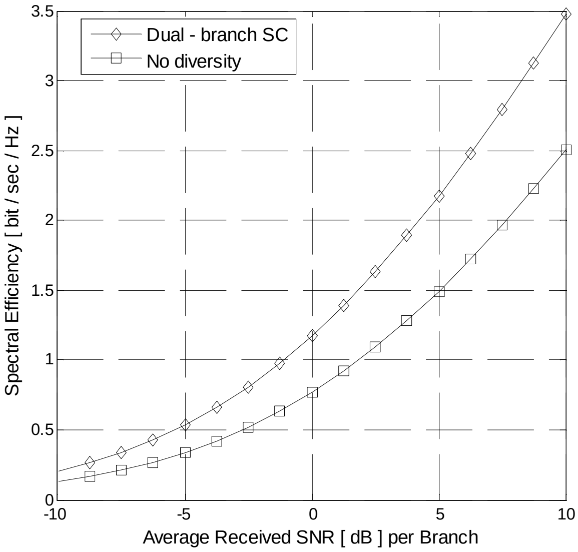

Figure 1. Spectral Efficiency for a worst case of fading versus average received SNR $\bar{\gamma}$ under ORA.

It is seen in Table. 1 that as the truncation error bounds becomes tighter as the truncation level, N, increases. It means that as the truncation level increases the difference between upper and lower bounds for each average received SNR per branch $\bar{\gamma}$ decreases and hence calculated spectral efficiency becomes more appropriate. That's why the infinite series in η_{ORA} has been truncated at the 15th term to calculate the spectral efficiency for the Fig. 1.

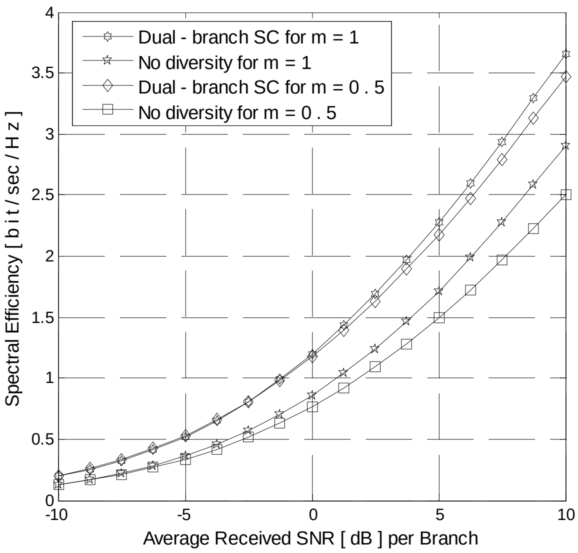

Figure 2. Spectral Efficiency versus average received SNR $\bar{\gamma}$ under ORA.

In Fig. 2, the spectral efficiency under ORA scheme is plotted as a function of the average received SNR per branch $\bar{\gamma}$. For comparison, the spectral efficiency of uncorrelated Rayleigh fading channels with dual-branch SC and without diversity, which was obtained in [8, Eq. (44)] and [8, Eq. (34)] respectively, is also presented in Fig. 2. As expected, as the channel fading conditions improves, i.e, m and / or $\bar{\gamma}$ increases, spectral efficiency improves. It is very interesting to observe that the spectral efficiency without diversity for $\bar{\gamma} \leq -7.5dB$ remains same as we move from worst fading condition to Rayleigh. Similarly, dual-branch SC for $\bar{\gamma} \leq -2.5\ dB$, gives almost identical performance even we move from worst fading condition to Rayleigh.

Fig. 3 states that spectral efficiency versus average received SNR per branch $\bar{\gamma}$ over Nakagami-0.5 fading channel remains zero as we go from no diversity to dual-branch SC under CIFR scheme.

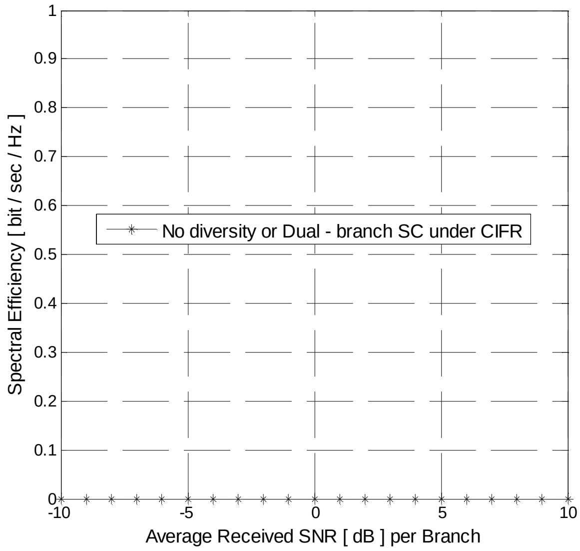

Figure 3. Spectral Efficiency for a worst-case of fading versus average received SNR $\bar{\gamma}$.

Table 1. Comparison of $\eta_{ORA,N}$, $\eta_{ORA,E-up}$, and $\eta_{ORA,E-low}$ at two different values of N for worst case of fading.

	$N=5$		
$\bar{\gamma}$ [dB]	$\eta_{ORA,N}$	$\eta_{ORA,E-up}$	$\eta_{ORA,E-low}$
-10	0.143708	0.061883	0.0581541295
-5	0.411463	0.120879	0.1153123659
0	0.977032	0.196274	0.1896116138
5	1.89534	0.27954	0.2723395591
10	3.10794	0.36565	0.3582797952
	$N=15$		
$\bar{\gamma}$ [dB]	$\eta_{ORA,N}$	$\eta_{ORA,E-up}$	$\eta_{ORA,E-low}$
-10	0.164232	0.041359	0.04064235544
-5	0.463140	0.069202	0.06833679633
0	1.07295	0.10041	0.09942977640
5	2.04205	0.13283	0.13180969046
10	3.30793	0.16564	0.164623141056

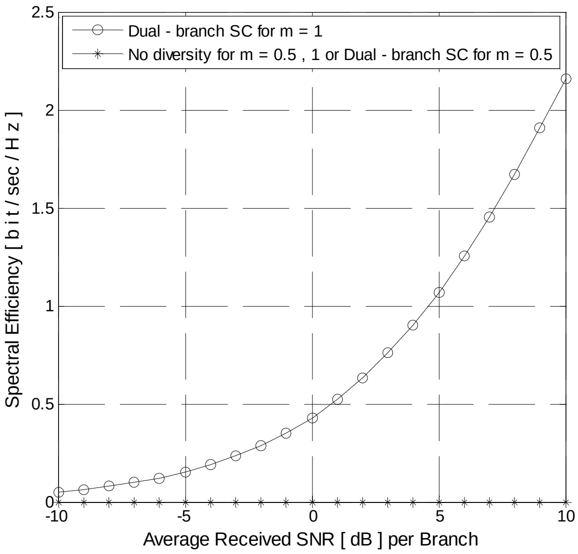

Figure 4. Spectral Efficiency versus average received SNR $\bar{\gamma}$ under CIFR.

In Fig. 4, the spectral efficiency under CIFR is plotted as a function of the average received SNR per branch $\bar{\gamma}$. For comparison, the spectral efficiency of uncorrelated Rayleigh fading channels with dual-branch SC, which was obtained in [8, Eq. (52)], is also presented in Fig. 4. It is observed that, as the channel fading conditions improve, i.e, m and / or $\bar{\gamma}$ increases, spectral efficiency with no diversity remains zero. However, employing a dual-branch SC system improves the channel capacity as we go from worst case of fading conditions to Rayleigh fading conditions.

In Fig. 5, the spectral efficiency of uncorrelated Nakagami-0.5 fading channels with and without diversity is plotted as a function of $\bar{\gamma}$, considering ORA, OPRA, and TIFR adaptation schemes with the aid of (6), (23), [17, Eq. (27)], [17, Eq. (35)], [15, Eq. (8)], and [15, Eq. (22)]. It shows that, the spectral efficiency with no diversity under ORA scheme improves over TIFR for $\bar{\gamma} \geq 5dB$. It is also observed that the spectral efficiency with dual-branch SC under ORA scheme improves over TIFR for $\bar{\gamma} \geq 0dB$ and OPRA scheme provides better efficiency under worst case of fading. It is also interesting to observe that for $\bar{\gamma} \leq -7.5dB$ ORA scheme with dual-branch SC gives inferior performance with respect to TIFR scheme without diversity.

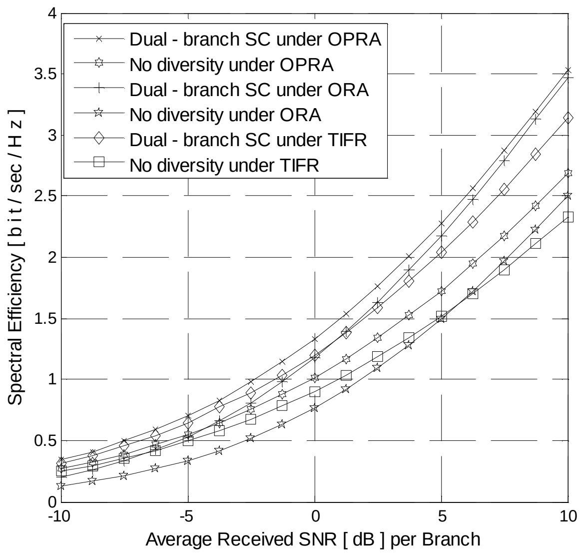

Figure 5. Spectral Efficiency versus average received SNR $\bar{\gamma}$ under different adaptive transmission schemes.

5. Conclusions

In this paper, closed-form expressions for the spectral efficiency of dual-branch SC and no diversity under ORA and CIFR schemes have been obtained and analyzed. Error bounds have been derived for truncated infinite series under ORA. Numerical results illustrate that the bounds can be used effectively to determine the number of terms needed to achieve desirable level of accuracy. Results have been plotted, which show that by increasing $\bar{\gamma}$ and/or employing diversity, spectral efficiency improves under ORA scheme. It is also observed that the spectral efficiency under CIFR is zero under worst case of fading even when a dual-branch SC is utilized. It is important to note that the spectral efficiency using ORA scheme remains almost same even when fading conditions improve from Nakagami-0.5 to Rayleigh either under dual-branch SC for $\bar{\gamma} \leq -2.5dB$ or under no diversity for $\bar{\gamma} \leq -7.5dB$. This paper finally concludes that under worst case of fading TIFR scheme is a better choice for low average received SNR and ORA scheme is for high average received SNR even employing diversity. Therefore it is recommended that under worst fading condition, ORA scheme is not always a better choice over TIFR.

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