Reliability and Failure Functions of the Consecutive *k*-out-of-*m*-from-*n*: F Linear and Circular System

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Abstract: The consecutive *k*-out-of-*m*-from-*n*: F system consists of *n* linearly or circularly ordered components, it fails if, and only if among any *m*-consecutive components, there is at least *k* failed components. In this paper, a new algorithm to find the reliability and the failure functions of the consecutive *k*-out-of-*m*-from-*n*: F linear and circular systems is obtained.

Keywords: Consecutive *k*-out-of-*n*: F system, Consecutive *k*-out-of-*m*-from-*n*: F system, Modular arithmetic, Equivalent relation.

1. Introduction

The consecutive *k*-out-of-*n*: F system models appeared in the last quarter of the last century, it had many generalizations and a wide applications area such as, the design of integrated circuits, microwave relay stations in telecommunications, oil pipeline systems, vacuum systems in accelerators, computer ring networks (*k* loop), and spacecraft relay etc., see [2-4], [7-8], [11-14], [17-18] and therein.

One of these generalization is the consecutive *k*-out-of-*m*-from-*n*: F system, where $1 \le k \le m \le n$, (sometimes called consecutive *k*-within *m*-out-of-*n*: F system or *k*-within-consecutive *m*-out-of-*n*: F system), it was formally introduced by Griffith [18] despite it has been mentioned by Tong [19]. It has two types, the linear and the circular systems; it consists of *n* linearly or circularly ordered components and fails if, and only if, there exist at least *k* failed components among any *m* consecutive components. When *k*=*m*, it is a consecutive *k*-out-of-*n*: F system, while if *m*=*n*, it is a *k*-out-of-*n*: F system. The system applied in many problems related to connection with tests for nonrandom clustering, quality control & inspection procedures, service systems, radar detection and the telecommunication system that uses *n*-bytes messages.

Sfakianakis et al. [10] studied the reliability of system when the components are *i.i.d*., Papastavridis and Koutras [15] provided an upper and lower bound of the reliability of the system, with unequal components failure probabilities, Habib and Szatai [1] improved these boundaries, Iyer [16] studied the distribution of the lifetime of the system with independent exponentially distributed components lifetime, while Higashiyama et al. [18] obtained a recursive algorithms for a linear and the circular types, for *k*=2, where the components are *s*-independent with unequal probabilities of failure, Malinowski and Preuss [5-6] studied the system and introduced a recursive algorithm for both, the linear and the circular system respectively.

In the second section, we recall and adapt the needed facts, and results from the set theory that is concerning relation between components, and used it to represent the consecutive *k*-out-of-*m*-from-*n*: F circular system, which is helpful to determine the properties of its failure states. Next, we provide a definition of the collection of all failure and functioning states, and compute the maximum number of failed components, whenever, the system is in the functioning state. In section 3, we make a projection of all relations and definitions of the circular type on to the linear one. In the fourth section, we proposed an algorithm to compute the reliability and the failure functions of the linear and circular consecutive *k*-out-of-*n*: F system, illustration example is provided also.

The following assumptions are assumed to be satisfied by all system bellow:

1. The state of the component and the system is either "functioning" or "failed"
2. All the components are mutually statistically independent.

Also, we provide below, the notations and definitions that will be used throughout the article.

L(C) : Linear (circular)

i.i.d. : Independent and identically distributed

I_j^i $=\{i, i+1, ..., j\}$ $\quad 1 \le i < j \le n$

$P(\mathrm{I}_n^1)$: The power set of I_n^1.

$X=\{x_1, x_2, ..., x_j\}$: a subset of I_n^1, such that $x_i < x_h$ for all $1 \le i < h \le j \le n$

f_n^{α} : The composite function α times, where $f_n(x) = x \bmod_n + 1 : x \in \mathrm{I}_n^1$

$d_X = (d_1^X, d_2^X, ..., d_j^X)$: The rotation of $X=\{x_1, x_2, ..., x_j\}$, where $d_i^X \ge 1$ is the minimum integer number such that $f_n^{d_i^X}(x_i) = x_{i+1}$, for *i*=1,2,…, *j*-1, and $f_n^{d_j^X}(x_j) = x_1$, such that $n = \sum_{i=1}^{j} d_i^X$.

d_X^r $=(d_{j-r+1}^X, ..., d_j^X, d_1^X, ..., d_{j-r}^X)$ $\quad r \in \mathbf{Z}$, where $d_X^0 = d_X^j, d_X^{j+s} = d_X^s$

$\overline{X}$: The complement of the set *X*.

$|X|$: The cardinality of the set *X*.

≡,~ : Equivalence relations

$i \underset{j}{\oplus} s$ $=(i+s) \bmod j$, if $i+s = nj$ then $i \underset{j}{\oplus} s = j$

$p_i(q_i)$:The reliability (unreliability) of the i^{th} components

$R(X)(F(X)) = p_X = \prod_{i \in \overline{X}} p_i \prod_{j \in X} q_j$, the reliability (unreliability) of the set *X*.

p_n^s $= p(n,s) = p^s q^{n-s}$

$[\overline{n}]$: The greatest integer number of *n*.

2. The consecutive *k*-out-of-*m*-from-*n*: F circular system

Consider the circular consecutive *k*-out-of-*m*-from-*n*: F system, I_n^1 denotes the indices of the components, $P\left(I_n^1\right)$ is the failure space of the components. The set $X=\{x_1,x_2,...,x_j\}\subseteq I_n^1$ represents the system and consists of all indices of the failed components (For example in the consecutive 2-out-of-3-from-6: F circular system, the set $X=\{2,4\}$, for simply, write it $X=24$, it means that the 2nd and the 4th components are only the failed component. Actually, X is a failure state, since; there are 2 failed components among the three consecutive components with indices 2, 3, and 4.

Nashwan [9] define a symmetric property of a circular system using a bijection function $f_n: I_n^1 \to I_n^1$, and partitioned $P\left(I_n^1\right)$ into finite mutually pairwise disjoint equivalence classes like $[X]=\{f_n^{\alpha}(X):\alpha\in\mathbf{Z}\}$, where $X\equiv Y$ if $f_n^{\alpha}(X)=Y$ for some $\alpha\in\mathbf{Z}$, where

1. Y is a rotation of X, and written as $X\sim Y$, if $Y\in P\left(I_n^1\right)$ such that $d_Y\in D(X)=\{d_X^r : r\in I_j^1\}$.
2. $X\equiv Y \Leftrightarrow X\sim Y$ i.e. $[X]=[Y]$.

If $X=\{x_1,x_2,...,x_j\}\subseteq I_n^1$ represents the consecutive *k*-out-of-*m*-from-*n*: F circular system, define $S_C^X=\{S_i^X : i\in I_j^1\}$ where $S_i^X=\sum_{t=0}^{k-2} d_{i\oplus_j t}^X$, d_i^X is the number of steps to walk on the circle from x_i to $x_{i\oplus_j 1}$, hence S_i^X is the total number of steps walk through k failed components (from x_i to $x_{i\oplus_j k-1}$), if $S_i^X\le m-1$, then there is at maximum *m*-1 steps (*m* consecutive components), in other words, *k* failed components among *m* consecutive components, which leads to the failure state of the system.

Definition 2.1: Let $X\in P\left(I_n^1\right)$ represents the consecutive *k*-out-of-*m*-from-*n*: F circular system, X is called a failed set if there exists $S_i^X\in S_C^X$, such that $S_i^X<m$. Vice versa, it is called a functioning set if $S_i^X\ge m$ for all $S_i^X\in S_C^X$.

In this context, the failure (functioning) space of the system $\Psi_C^{k,m,n}\left(\Theta_C^{k,m,n}\right)$ respectively is the collection of all failed (functioning) sets.

Lemma 2.1: Consider the consecutive *k*-out-of-*m*-from-*n*: F circular system, $X,Y\in P\left(I_n^1\right)$ such that $Y\in[X]$

1. If $X\in\Psi_C^{k,m,n}\left(\Theta_C^{k,m,n}\right)$, then $Y\in\Psi_C^{k,m,n}\left(\Theta_C^{k,m,n}\right)$.
2. $R(Y)=F(Y)=p_{f_n^{\alpha}(X)}$ for some $\alpha\in\mathbf{Z}$.
3. If the components are *i.i.d.*, then $R(Y)=R(X)=p_n^{|X|}$ and $F(Y)=F(X)=p_n^{|X|}$.

Proof:

1. Assume $X\in\Psi_C^{k,m,n}$, then there exists $S_i^X\in S_C^X$, such that $S_i^X<m$, let $Y\in[X]$, then there exists $r\in I_j^1$ such that, $d_Y=d_X^r$, then $S_C^Y=S_C^X$, i.e. there exists $S_i^Y\in S_C^Y$, such that $S_i^Y<m$, then Y is a failed set. On the contrary, if $X\in\Theta_C^{k,m,n}$, then $S_i^X\ge m$ for all $S_i^X\in S_C^X$, and since $Y\in[X]$, there exists $r\in I_j^1$ such that, $d_Y=d_X^r$, then $S_C^Y=S_C^X$, then then for all $S_i^Y\in S_C^Y$, $S_i^Y\ge m$, hence $Y\in\Theta_C^{k,m,n}$.
2. If $Y\in[X]$, there exists $\alpha\in\mathbf{Z}$, such that $Y=f_n^{\alpha}(X)$ $R(Y)=p_Y=p_{f_n^{\alpha}(X)}$. Also $F(Y)=p_Y=p_{f_n^{\alpha}(X)}$
3. If the components are *i.i.d.*, then $|X|=|Y|$, apply 2, $R(X)=p_n^{|X|}=p_n^{|Y|}=R(Y)$.

Example 2.1: $\{1245\}\in\Psi_C^{3,4,9}$, since $S_1^{\{1245\}}=S_2^{\{1245\}}=3<4$ and $S_C^{\{1245\}}=\{3,6\}$, moreover, $[1245]=\{1245,....,1259\}\in\Psi_C^{3,4,9}$. While, $\{1257\}\in\Theta_C^{3,4,9}$, since $S_C^{\{1257\}}=\{4,5\}$ and all $S_i^X\in S_C^X$, such that $S_i^X\ge 4$, $[1257]=\{1257,2368,...,1469\}\in\Theta_C^{3,4,9}$.

Notes:

1. According lemma 2.1, if $X\in\Psi_C^{k,m,n}\left(\Theta_C^{k,m,n}\right)$, $[X]$ is called failed (functioning) class, since all set $Y\in[X]$ is failed (functioning) set.
2. Using lemma 2.1, classify these classes for a failed or functioning classes, hence, $\Psi_C^{k,m,n}=\bigcup_{i=1}^{s}[X_i]$ and $\Theta_C^{k,m,n}=\bigcup_{t=s+1}^{r}[X_t]$, then $P\left(I_n^1\right)=\Psi_C^{k,m,n}\cup\Theta_C^{k,m,n}$.
3. Define the reliability of these classes is $\mathrm{R}[X]=\sum_{Z\in[X]}R(Z)=\sum_{Z\in[X]}p_Z$, and the failure function is $\mathrm{F}[X]=\sum_{Z\in[X]}F(Z)=\sum_{Z\in[X]}p_Z$.

Lemma 2.2: In the functioning consecutive *k*-out-of-*m*-from-*n*: F circular system, the maximum possible number of failed components is $M_C=(k-1)\lceil n/m\rceil+s_C$ where

$$s_C=\begin{cases} b-m+k-1 & b\ge m-k+1 \\ 0 & \text{otherwise}\end{cases} \quad : b=n \bmod m$$

Proof: Let $X=\{x_1,...,x_{M_C}\}$ represents the system, divide the circle by arcs, each arc consists of m consecutive components. To reach the maximum number of failed components on the circle that keeping the system in the functioning state, each arc has at maximum *k*-1 failed components, such that, the last failed component in any arc is away from the 1st failed component in the next arc is at least *m*- *k*+1 functioning components, otherwise the system fails. WLOG assume the *k*-1 failed components always located from 1st to the *k*-1th components in every arc.

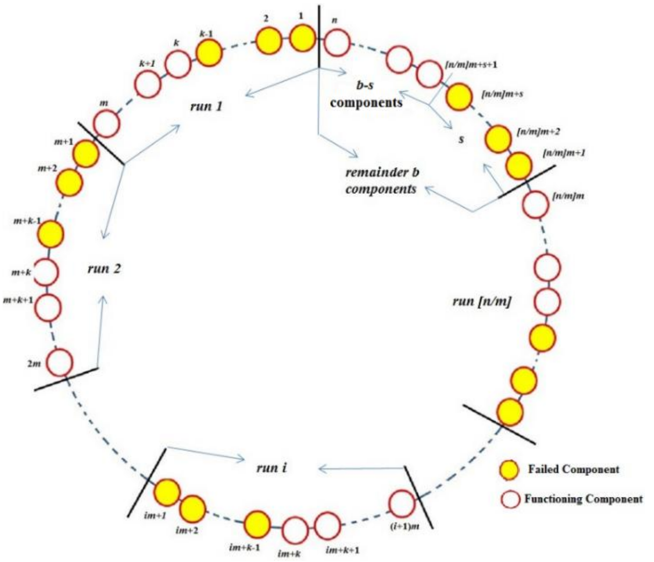

Figure 1. The functioning consecutive *k*-out-of-*m*-from-*n*: F circular system.

If *m* divides *n*, then we have $\overline{\lceil n/m \rceil}$ arcs, i.e. $M_C = (k-1)\overline{\lceil n/m \rceil}$, otherwise $n = \lceil n/m \rceil m + b$ where $b = n \bmod_m < m$, which implies that, $b = n - \lceil n/m \rceil m < m$, then $b - m + (k-1) < k-1$, see figure 1.

$$d_X = \left(\overbrace{\underbrace{1,\ldots,1}_{k-2},d_{k-1},\underbrace{1,\ldots,1}_{k-2},d_{k-1},\ldots,\underbrace{1,\ldots,1}_{k-2},d_{k-1}}^{\lceil n/m \rceil \text{ times}},\underbrace{d^X_{M-s+1},\ldots\ldots,d^X_{M-1},d^X_M}_{s_C} \right)$$

Let s_C the number of failed components in the remainder *b* components, then $s_C \le k-1$ failed components, otherwise the system fails, Note that $M_C = \lceil n/m \rceil (k-1) + s_C$ and the distance from the 1st component in the 1st arc is far away from the last failed component in the remainder components is at least *m*-*k* +1 components, otherwise the system fails. i.e. $b - s_C \ge m - k + 1 \Rightarrow \; b - m + k - 1 \ge s_C$, which implies that
$$s_C = \begin{cases} b - m + k - 1 & b \ge m - k + 1 \\ 0 & b < m - k + 1 \end{cases}$$

3. The consecutive *k*-out-of-*m*-from-*n*: F linear system

The difference between the structure of the linear and the circular consecutive *k*-out-of-*m*-from-*n*: F system is the connection between the 1st and the *n*th components, which causes more failure states than that in the linear system, i.e. $\Psi_L^{k,m,n} \subseteq \Psi_C^{k,m,n}$. In this context, if $X \in P\left(\mathrm{I}_n^1\right)$ represents the consecutive *k*-out-of-*m*-from-*n*: F linear system, and $S_L^X = \left\{ S_i^X : i \in \mathrm{I}^1_{j-(k-1)} \right\}$ where $S_i^X = \sum_{t=0}^{k-2} d^X_{i+t}$. The system fails if there exists $S_i^X \in S_L^X$, such that $S_i^X < m$, where condition $i \in \mathrm{I}^1_{j-(k-1)}$ removes the effects of the connection between the 1st and the *n*th components, *Recall example 2.1*, $\{2389\} \in [1245] \in \Psi_C^{3,4,9}$, while $\{2389\} \notin \Psi_L^{3,4,9}$ since $d_{2389} = (1,5,1,2)$ and $S_L^{\{2389\}} = \{6\} > 4$.

In the next lemmas, our mission is to omit the extra failures states from $\Psi_C^{k,m,n}$ to achieve $\Psi_L^{k,m,n}$.

Lemma 3.1: Consider the consecutive *k*-out-of-*m*-from-*n*: F linear system in the functioning state, then the maximum number of failed component is $M_L = (k-1)\overline{\lceil n/m \rceil} + s_L$ where $s_L = \begin{cases} k-1 & b \ge k-1 \\ b & b < k-1 \end{cases}$ and $b = n \bmod m$.

Proof: Use the same technique of the proof in lemma 3.2, and open the circle between the 1st and the *n*th components, but the remainder has at maximum any *k*-1 failed components. Hence the remainder $b = n \bmod m$ at maximum has *k*-1 failed components if $b > k-1$, otherwise $b < k-1$, hence no matter the number of failed components in the remainder.

Lemma 3.2: Let $X = \left\{ x_1,\ldots,x_j \right\} \in \Psi_C^{k,m,n}$ represents the consecutive *k*-out-of-*m*-from-*n*: F circular system, then $X \in \Theta_L^{k,m,n}$, if *X* holds the following boundary conditions:

1. There exists $S_i^X < m$ for any $i \in \mathrm{I}_j^{j-(k-2)}$.

All $S_i^X \ge m$ for all $i \in \mathrm{I}^1_{j-(k-1)}$.

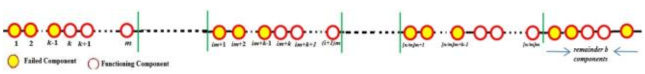

Figure 2. The functioning consecutive *k*-out-of-*m*-from-*n*: F linear system.

4. The proposed algorithm of the reliability and failure functions of the consecutive *k*-out-of-*m*-from-*n*: F linear & circular systems

If *j* is the total number of failed components in the consecutive *k*-out-of-*m*-from-*n*: F linear and circular system, then

1. For *j*=0,1, …, *k*-1, all states are functioning states, then $\mathrm{R}_j^C = \binom{n}{j} p_n^{n-j}$ and $\mathrm{F}_j^C = 0$.
2. For each *j*=*k*,*k*+1,…, M_C , determine the rotations $\left(d_1,d_2,\ldots,d_j\right)$ such that $n = \sum_{i=1}^{j} d_i$, and find the corresponding set *X* and $[X] = \left\{ f_n^{\alpha}(X) : \alpha \in \mathbf{Z} \right\}$.
3. Compute $S_C^X = \left\{ S_i : i \in \mathrm{I}_j^1 \right\}$.
4. If there exists $S_i \in S_C$, where $S_i < m$, then $[X] \in \Psi_C^{k,m,n}$, otherwise $[X] \in \Theta_C^{k,m,n}$.
5. For the linear system, separate all states from $\Psi_C^{k,m,n}$ that do not hold the boundary conditions of lemma 3.2. and 3.1., and add it to $\Theta_L^{k,m,n}$.

6. For each $j > M_{L(C)}$ all states are a failure, then

$$F_j^C = \binom{n}{j} p_n^{n-j} \text{ and } R_j^C = 0$$

7. Find $F_j^{L(C)}\left(R_j^{L(C)}\right)$, which is the failure (reliability) summation of the classes $[X] \in \Psi_{L(C)}^{k,m,n}\left(\Theta_{L(C)}^{k,m,n}\right)$, where $|X| = j$.

8. The reliability function is $R_{L(C)} = \sum_{j=0}^{M_{L(C)}} R_j^{L(C)}$, while the failure function is $F_{L(C)} = \sum_{j=k}^{n} F_j^{L(C)}$.

Example 4.1: Find the failure and the functioning functions of the *i.i.d.* consecutive 3-out-of-4-from-8: F linear and circular system $M_{L(C)} = (k-1)\overline{[n/m]} + s_{L(C)} = 2\times 2 + 0 = 4$

For j=3

$(1,1,6) = d_{123} \Rightarrow$

$[123] = \{123,234,345,456,567,678,\underline{178},\underline{128}\} \in \Psi_C^{3,4,8}$

$\{178,128\} \in \Theta_L^{3,4,8}$

$(1,2,5) = d_{124} \Rightarrow$

$[124] = \{124,235,346,457,568,167,\underline{278},\underline{138}\} \in \Psi_C^{3,4,8}$

$\{278,138\} \in \Theta_L^{3,4,8}$

$(1,3,4) = d_{125} \Rightarrow$

$[125] = \{125,236,347,458,156,267,378,148\} \in \Theta_{L(C)}^{3,4,8}$

$(1,4,3) = d_{126} \Rightarrow$

$[126] = \{126,237,348,145,256,367,478,158\} \in \Theta_{L(C)}^{3,4,8}$

$(1,5,2) = d_{127} \Rightarrow$

$[127] = \{\underline{127},\underline{238},134,245,356,467,578,\underline{168}\} \in \Psi_C^{3,4,8}$

$\{127,238,168\} \in \Theta_L^{3,4,8}$

$(2,2,4) = d_{135} \Rightarrow$

$[135] = \{135,246,357,468,157,268,137,248\} \in \Theta_{L(C)}^{3,4,8}$

$(2,3,3) = d_{136} \Rightarrow$

$[136] = \{136,247,358,146,257,368,147,258\} \in \Theta_{L(C)}^{3,4,8}$

$F_3^C = 24p_8^5 \quad R_3^C = 32p_8^5 \quad F_3^L = 17p_8^5 \quad R_3^L = 39p_8^5$

For j=4

$(1,1,1,5) = d_{1234} \Rightarrow$

$[1234] = \{1234,2345,3456,4567,5678,1678,\underline{1278},1238\} \in \Psi_C^{3,4,8}$

$\{1278\} \in \Theta_L^{3,4,8}$

$(1,1,2,4) = d_{1235} \Rightarrow$

$[1235] = \{1235,2346,3457,4568,1567,2678,\underline{1378},1248\} \in \Psi_C^{3,4,8}$

$\{1378\} \in \Theta_L^{3,4,8}$

$(1,1,3,3) = d_{1236} \Rightarrow$

$[1236] = \{1236,2347,3458,1456,2567,3678,\underline{1478},\underline{1258}\} \in \Psi_C^{3,4,8}$

$\{1478,1258\} \in \Theta_L^{3,4,8}$

$(1,1,4,2) = d_{1237} \Rightarrow$

$[1237] = \{1237,2348,1345,2456,3567,4678,1578,\underline{1268}\} \in \Psi_C^{3,4,8}$

$\{1268\} \in \Theta_L^{3,4,8}$

$(1,2,1,4) = d_{1245} \Rightarrow$

$[1245] = \{1245,2356,3467,4578,1568,\underline{1267},\underline{2378},1348\} \in \Psi_C^{3,4,8}$

$\{1267,2378\} \in \Theta_L^{3,4,8}$

$(1,2,2,3) = d_{1246} \Rightarrow$

$[1246] = \{1246,2357,3468,1457,2568,1367,\underline{2478},\underline{1358}\} \in \Psi_C^{3,4,8}$

$\{2478,1358\} \in \Theta_L^{3,4,8}$

$(1,2,3,2) = d_{1247} \Rightarrow$

$[1247] = \{1247,2358,1346,2457,3568,1467,2578,\underline{1368}\} \in \Psi_C^{3,4,8}$

$\{1368\} \in \Theta_L^{3,4,8}$

$(1,3,1,3) = d_{1256} \Rightarrow [1256] = \{1256,2367,3478,1458\} \in \Theta_{L(C)}^{3,4,8}$

$(1,3,2,2) \Rightarrow$

$[1257] = \{\underline{1257},\underline{2368},1347,2458,1356,2467,3578,1468\} \in \Psi_C^{3,4,8}$

$\{1257,2368\} \in \Theta_L^{3,4,8}$

$(2,2,2,2) = d_{1357} \Rightarrow [1357] = \{1357,2468\} \in \Theta_{L(C)}^{3,4,8}$

$F_4^C = 64p_8^4 \quad R_4^C = 6p_8^4 \quad F_4^L = 52p_8^4 \quad R_4^L = 18p_8^4$

For j=5, 6, 7, 8 all states are in the failure states, hence $F_j^{L(C)} = \binom{8}{j} p_8^{8-j}$, and $R_j^C = 0$.

$$F_C = \sum_{j=3}^{8} F_j^C = 24p_8^5 + 64p_8^4 + 56p_8^3 + 28p_8^2 + 8p_8^1 + p_8^0$$

$$R_C = \sum_{j=0}^{n-k+1} R_j^C = p_8^8 + 8p_8^7 + 28p_8^6 + 32p_8^5 + 6p_8^4$$

$$F_L = \sum_{j=3}^{8} F_j^L = 24p_8^5 + 64p_8^4 + 56p_8^3 + 28p_8^2 + 8p_8^1 + p_8^0$$

$$R_L = \sum_{j=0}^{n-k+1} R_j^L = p_8^8 + 8p_8^7 + 28p_8^6 + 40p_8^5 + 18p_8^4$$

5. Conclusions

This paper introduced an algorithm to find the reliability and the failure function of the consecutive *k*-out-*m*-from-*n*: F linear and circular systems, in this context, the algorithm determines the failure and the functioning space of the system (all failure and functioning states), where, the failure space of the linear system is a sub collection of the circular one. Moreover, the proposed algorithm computes the maximum number of failed components, whenever, the system is in the functioning state, whether in the linear or circular type.

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