

Low Complexity Joint Sub-Carrier Pairing, Allocation and Relay Selection in Cooperative Wireless Networks

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Abstract: Multi-carrier cooperative relay-based wireless communication is of particular interest in future wireless networks. In this paper we present resource allocation algorithm in which sub-carrier pairing is of particular interest along with fairness constraint in multi-user networks. An optimization of sub-carrier pair selection is formulated through capacity maximization problem. Sub-carrier pairing is applied in both two-hop Amplify & Forward (AF) and Decode & Forward (DF) cooperative multi-user networks. We develop a less complex centralized scheme for joint sub-carrier pairing and allocation along with relay selection. The computational complexity of the proposed algorithms has been analyzed and performance is compared with Exhaustive Search Algorithm.

Keywords: Relay Networks, OFDM, Resource Block, Sub-carrier Pairing, Relay Protocols, Relay Network.

1. Introduction

Relay-based cooperative communication has been proposed in the cellular networks to improve reliability, enhance capacity, reduce total power consumption and extend coverage area [1-5]. The deployment of relay terminal in Orthogonal Frequency Division Multiplexing (OFDM)-based wireless cellular networks is a promising design to meet high data rates and extended coverage demands in future wireless networks. However, the presence of relay terminals makes the issue of resource allocation more challenging and considerable.

Relay based cooperative wireless communication is a hot research topic now a day. Recently, several results on resource allocation in cooperative relaying networks have been reported in the literature [6-8].

The sub-carriers are the most important resources in the OFDM-based transmission and the allocation of these to different users according to channel conditions has already been proven to provide significant gain in system efficiency under fading wireless environment [3]. In most of the literature, it is assumed that the same sub-carrier is used in both the first and the second hops of transmission [9, 10]. But due to independent channel fading on the same sub-carrier over the two hops, the system performance may not be optimal. The system performance can be enhanced further by sub-carrier pairing in the two hops according to their

channel conditions [11]. In [11], sub-carrier pairing based resource allocation for cooperative multi-relay networks is addressed for Amplify & Forward (AF) protocol. In [12], the concept of sub-carrier pairing in relay networks was introduced in the three-node network using the Decode & Forward (DF) protocol. In [13], resource allocation with sub-carrier pairing is investigated under a joint sum-power constraint for both AF and DF systems. Symbol Error Performance (SEP) analysis with sub-carrier pairing in OFDM relaying systems is presented in [14]. In all of these papers, the network of interest is a single source-destination pair with either single relay or multiple relays. But in practical scenarios, interference due to multiple users also plays a critical role in system performance degradation [15].

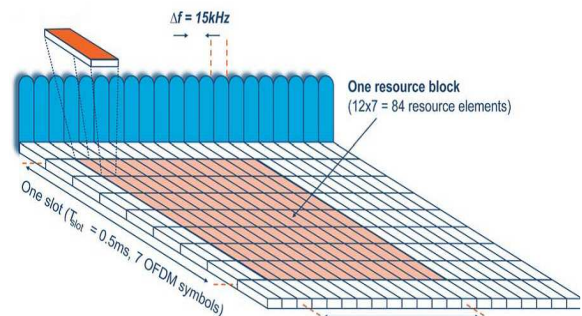


Figure 1. LTE Downlink Physical Resource [16]

In [17] and [18], Sub-carrier pairing based resource allocation has been addressed in Two-Way Relaying (TWR) networks with AF protocol only. In this paper we propose a new Low Complexity Iterative RB-Pairing and Allocation (LIRBPA) algorithm in multiuser multi-relay One-way Relaying (OWR) networks with proportional fairness among users.

In Long Term Evolution (LTE) system, Resource block (RB) is the minimal unit to be allocated as shown in Fig. 1. A single RB consists of twelve consecutive OFDM sub-carriers [19]. OFDM uses a large number of sub-carriers having smaller bandwidth for multi-carrier transmission. The basic LTE downlink physical resource grid is shown in Fig. 1. In the frequency domain, the spacing between the sub-carriers

(Δf) is 15 kHz. Each resource block has a total size of 180 kHz in the frequency domain and 0.5ms in the time domain. Each user is allocated a number of resource blocks in the time–frequency grid. The allocation of resource blocks to users depends on the scheduling mechanisms in the frequency and time dimensions [16].

The rest of the paper is divided into five additional sections. Section 2 describes the system model and basic assumptions. The problem formulation and description is presented in Section 3 while low complexity RB-pairing and allocation algorithm is presented in Section 4. Furthermore, numerical results with simulation are illustrated in Section 5. Finally, conclusion is provided in Section 6.

2. System Model

In this paper we consider a two-hop multi-user multi-relay-assisted cooperative wireless network that consists of M mobile terminals (MTs), R relay terminals (RTs) and a base station (BS) as shown in Fig. 2. A downlink transmission is considered where MTs receive information from the BS through the RTs. Each RT operates in a time-division half-duplex mode using AF protocol.

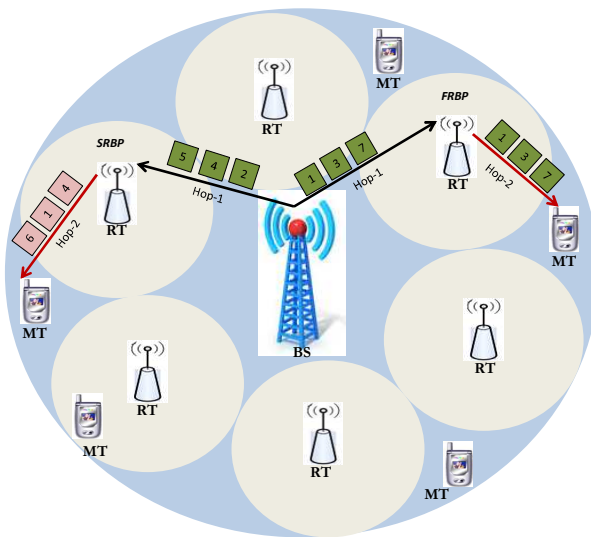


Figure 2. Multi-Users Multi-Relay Cooperative Network

By considering multi-carrier transmission, it is assumed that K RBs are available. In the rest of the paper, the term RB is used instead of a sub-carrier. The frequency selective fading channels are assumed where each RB experiences independent channel environment while all sub-carriers in a single RB experience the same channel environment.

It is also assumed that there is no intra-cell interference due to orthogonal sub-carriers and perfect synchronization in OFDM based cooperative wireless system. However, in case of imperfect synchronization, the multi-user interference may occur due to the different carrier frequency offsets

between different MTs. This multi-user interference can easily be mitigated using some cancellation techniques such as the one proposed in [20].

3. Problem Formulation and Description

Let the signal received at RT on the RB index k is forwarded to MT over the RB index k' . Here, the RB index k' may not be the same as k and they form a RB-pair (k, k') . The total achieved network throughput (R_s) can be expressed as

$$R_s = \sum_{m=1}^M \sum_{r=1}^R \sum_{k=1}^K \sum_{k'=1}^K \frac{1}{2} \log_2 (1 + \delta^{k,k'} \alpha_{r,m}^{k,k'} \gamma_{r,m}^{k,k'}) \quad (1)$$

Here $\gamma_{r,m}^{k,k'}$ is the signal-to-noise ratio (SNR) for AF or DF protocol, as given in [1], while binary variables $\delta^{k,k'}$ and $\alpha_{r,m}^{k,k'}$ are RB-pairing index and RB-allocation index, respectively, and are given as:

$$\delta^{k,k'} = \begin{cases} 1 & \text{If RB } k \text{ is paired with RB } k' \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

$$\alpha_{r,m}^{k,k'} = \begin{cases} 1 & \text{If RB pair } (k,k') \text{ is allocated to MT } m \text{ at RT } r \\ 0 & \text{Otherwise} \end{cases}$$

Considering joint RB and power allocation with RB-pairing, an optimization problem is formulated in this sub-section. The main goal of this optimization problem is to maximize the overall system throughput given in (1):

$$\text{Maximize } \sum_{m=1}^M \sum_{r=1}^R \sum_{k=1}^K \sum_{k'=1}^K \frac{1}{2} \log_2 (1 + \delta^{k,k'} \alpha_{r,m}^{k,k'} \gamma_{r,m}^{k,k'}) \quad (3)$$

Subject to:

$$\sum_{k=1}^K \delta^{k,k'} = 1 \quad \forall k' \quad \text{and} \quad \sum_{k'=1}^K \delta^{k,k'} = 1 \quad \forall k \quad (4)$$

$$\sum_{m=1}^M \alpha_{r,m}^{k,k'} \leq 1, \quad \alpha_{r,m}^{k,k'} \in \{0,1\} \quad \forall k, \forall k' \quad (5)$$

$$\sum_{r=1}^R \alpha_{r,m}^{k,k'} \leq 1, \quad \alpha_{r,m}^{k,k'} \in \{0,1\} \quad \forall k, \forall k' \quad (6)$$

$$\sum_{r=1}^R \sum_{k=1}^K \sum_{k'=1}^K \delta^{k,k'} \alpha_{r,m}^{k,k'} \gamma_{r,m}^{k,k'} \geq \tau_m \quad \forall m \quad (7)$$

In the above, the constraint in (4) shows the RB-pairing and it ensures that each RB in 1st hop is paired with only one RB in 2nd hop. The constraint in (5) and (6) ensure that there is no intra-cell interference by assigning each RB-pair to only

one MT and RT. The relay selection is also achieved with (6), while (7) guarantees the minimum rate requirement for all MTs. Here τ_m in (5) is the MRR for each m^{th} MT.

The RB-pairing and allocation can be investigated by doing exhaustive search on all possible combinations of RB-pairs on all RTs for each user. The computational complexity of Exhaustive Search Algorithm (ESA) as proposed in [21] with multi-relay network can roughly be approximated to $O(M \times R \times K^K)$. With increasing number of MTs, RTs and RBs the complexity becomes too high which need to be avoided in practical applications. Therefore in the next section we will propose a new low complexity algorithm.

4. The Low-complexity Iterative RB-Pairing and Allocation (LIRBPA) Algorithm

To reduce the complexity of ESA we solve RB-pairing problem by using a one-to-one optimization solver known as Hungarian Algorithm (HA). The HA [22] is a one-to-one optimization solver for assignment problems with polynomial complexity. It has already been used in different resource allocation algorithms in non-relaying networks [23, 24]. Let us first we define a demand metric ($D_{r,m}^{k,k'}$) for m^{th} MT on RB-pair (k, k') as the achievable rate using r^{th} RT.

The following sub-steps are involved in LIRBPA.

1. The $K \times K$ matrix as shown in Fig.3 is established in such a way that the demand metric on each RB-pair is calculated as the maximum of $R \times M$ links.

$$D_{r,m}^{k,k'} = \max \{ r_{r,m}^{k,k'} \} \quad (8)$$

Here $r_{r,m}^{k,k'}$ indicates the rate achieved by m^{th} MT on RB-pair (k, k').

2. By applying HA on each $K \times K$ matrix as shown in Fig. 3 the best RB-pairing is achieved here.
3. The allocation of these RB-pairs is made iteratively to MTs by implementing the constraint given in (7).
4. The rows and the columns with assigned RB-pairs are eliminated.
5. The MTs satisfying MRR are removed temporarily.
6. 1-5 are repeated until all RBs are assigned or all MTs achieved MRR.
7. If RBs are still available the step 1-2 is repeated once to assign all remaining RBs to the best users to maximize the system throughput.

The Computational Complexity has been reduced significantly as compared to ESA. The total complexity of step 1 is $O(R \times M \times K)$ while the polynomial complexity of one iteration of HA is $O(K_n^3)$ [25], where K_n is the

number of unassigned RBs. We need maximum of K iterations to implement RB pairing and fairness in terms of data rate among all MTs, therefore the maximum complexity of step 1 is $O(R \times M \times K^2)$ and applying HA with ensuring fairness is $O(K^4)$. If step 7 exits then the complexity of this step is $O(K^3)$. The complexity of the whole LIRBPA algorithm is loosely upper-bounded as $O(R \times M \times K^6 + K^3)$. Considering practical scenario where $K \gg R$ and $K \gg M$, there is a significant reduction in the computational complexity with same outputs as comparing with the ESA algorithm.

		RBs in the 1 st Hop				
		1	2	K
RBs in the 2 nd Hop	1	D ^{1,1}	D ^{1,2}	D ^{1,K}
	2	D ^{2,1}	D ^{2,2}	D ^{2,K}

	K	D ^{K,1}	D ^{K,2}	D ^{K,K}

Figure 3. Snapshot of HA-Matrix for RB-Pairing

It is interesting to note that there is no scarification or trade off in throughput performance for low complexity by implementing LIRBPA algorithm. The proposed algorithm provides same throughput performance as with ESA with much less computational complexity, which is the main contribution in this paper.

5. Performance Evaluation

In this section we evaluate the performance of the proposed resource allocation algorithm with some simulation results.

5.1 Simulation Models and Parameters

In these simulations we assume random distribution of all RTs and MTs. We consider that all channels remain constant for one complete transmission while all noise variances are identical. To establish centralized resource allocation techniques it is assumed that CSI is known to the BS which is already commonly used assumption.

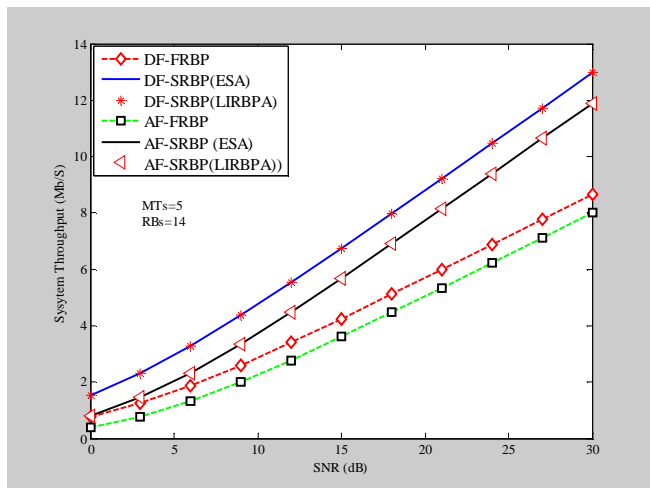
The Line-of-Sight (LOS) path loss model is used for BS-RT link as we assume that relays are in LOS of BS which has directional antennas for transmission. The Non Line-of-Sight (NLOS) path loss model is used for RT-MT links. Both path loss models follow those defined for the Urban Micro (UMi) environment for the evaluation of 4G mobile wireless systems [26]. Other simulation parameters are given in Table 1.

Table 1. Simulation Parameters

Parameter	Value
Cell Radius	1 Km
Carrier Frequency	2 GHz
OFDM Sub-carrier Bandwidth	15 KHz
Number of Sub-carrier per RB	12
Nominal Average SNR	20 dB

5.2 Simulation Results and Discussions

In the simulation results, we use the notations selective order RB-pairing (SRBP) for RB-pairing and Fixed order RB-pairing (FRBP) for No-RB-pairing, respectively.

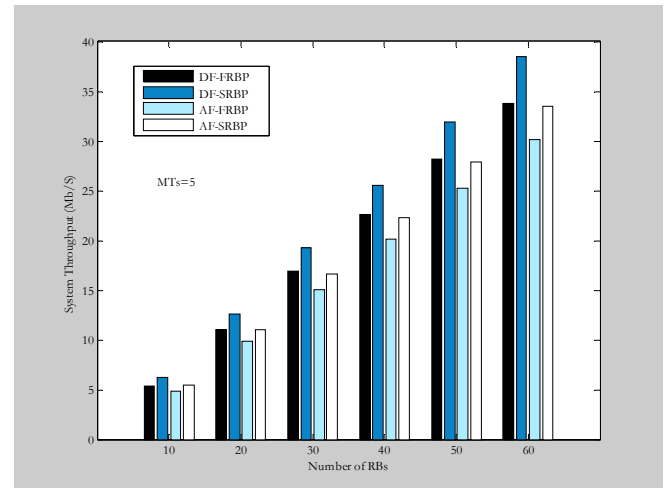
**Figure 4.** Achieved System Throughput versus SNR

The SRBP provides significant gain in system throughput over FRBP in both AF and DF systems. It can also be observed that the difference between two curves increases with an increase in SNR.

The performance of system throughput against average received SNR is depicted in Fig. 4. We simulate both ESA and LIRBPA for RB-pairing and allocation against different SNR values. It produces same result but with different computation complexity as described in previous section.

The result presented in Fig. 4 clearly confirms that RB-pairing provides significant gain in system throughput as compared to the system throughput when there is no RB-pairing used and data is transmitted on the same order RB in the 1st and 2nd hop, respectively. The figures shows the performance of system throughput at different nominal average SNR= $P_T^{k,k'} / \sigma^2$ values.

Fig. 5 shows the total achieved throughput for AF and DF systems with SRBP and FRBP against increasing number of available RBs. In both AF and DF systems MRR constraint is implemented. The better performance in term of throughput is achieved with SRBP than FRBP RB-pairing. It is also noticeable that the gain in performance with RB-pairing increases as the number of available RBs increases. Because the more RBs, the more flexibility that the system has in exploiting the channel diversity gains.

**Figure 5.** System Throughput versus Number of RBs

We also employ Jains fairness index [27] to assess the performance of our proposed algorithms. Jain's fairness index has already been widely used to determine the proportional fairness among users [25, 28]. It is given by

$$f = \frac{\left[\sum_{i=1}^M r_i \right]^2}{M \sum_{i=1}^M r_i^2} \quad (9)$$

where r_i is the normalized rate for the i^{th} user. The value of this index ranges from 0 (worst case) to 1 (best case). It is clear in Fig. 6 and Fig 7 that fairness index remains around 1 in both cases with different number of available RBs having fixed number of MTs and with different number of MTs having fixed number of RBS, respectively, which means that we are achieving maximum fairness among users in both cases. The only difference arises which is obvious, is that fairness index is increasing towards unity when we have fixed number of MTs and increasing number of RBs while it decrease from to away from unity in the other case.

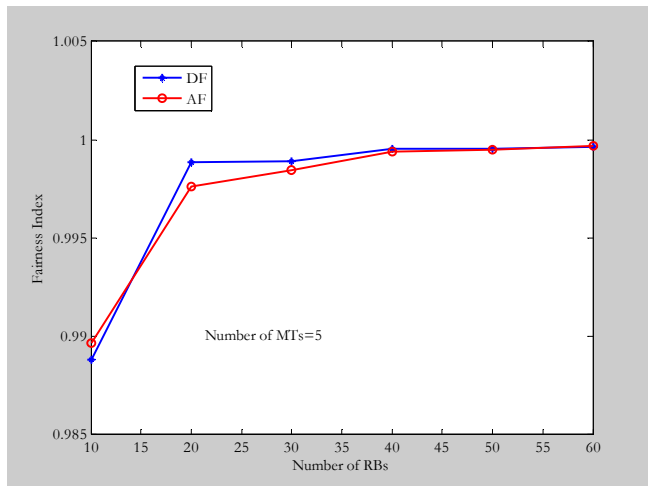


Figure 6. Variations of the Fairness Index against the Number of RBs

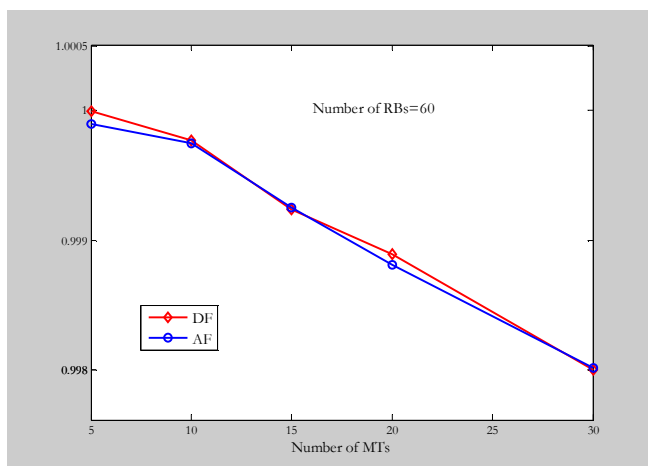


Figure 7. Variations of the Fairness Index against the Number of MTs

Fig. 8 and Fig. 9 show the bar graphs of RBs which are assigned to different MTs using SRBP and FRBP in AF and DF protocols respectively. Each bar position shows that how different RBs are paired in 1st and 2nd hops in different schemes. The SRBP is clearly shown in Fig. 8(a) and Fig. 9(a), in which 1st hop RBs are paired with different order RBs in 2nd hop, while Fig. 8(b) and Fig. 9(b) indicate that RB in 1st hop is paired with the same order RB in 2nd hop. The different color of bars indicates the different MTs.

6. Conclusions

In this paper we focus on sub-carrier pairing in dual-hop relay networks. We presented resource allocation algorithm in which sub-carrier pairing and allocation is jointly proposed with relay selection and fairness constraint in multi-user relay networks. We observe that the computational complexity of conventional Exhaustive Search Algorithm is too high and is not applicable in practical applications. Therefore we propose Hungarian

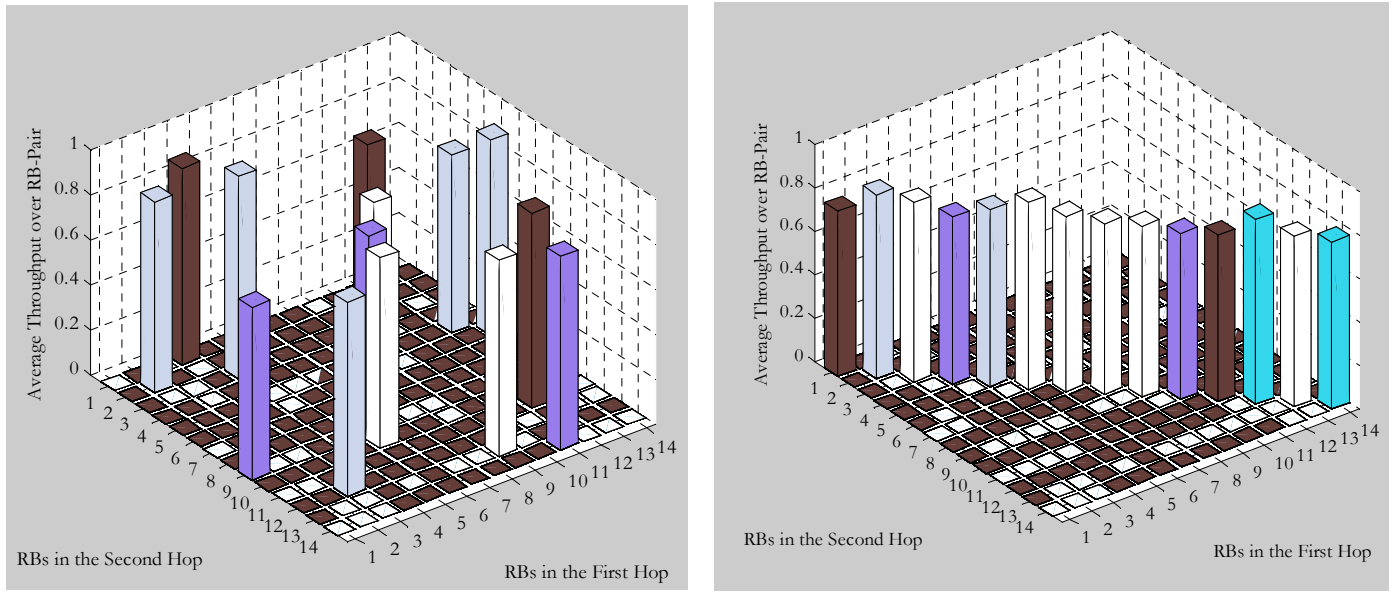
Algorithm based new low complexity iterative RB-pairing and allocation scheme which has much less computational complexity with same output performance.

Simulation results demonstrate that RB-pairing proposed provides significant gain in system throughput. The proposed algorithm is also capable to provide maximum fairness in terms of data rate among all users. The simple model and low computational complexity make the proposed algorithm suitable for solving such optimization problem in relay networks.

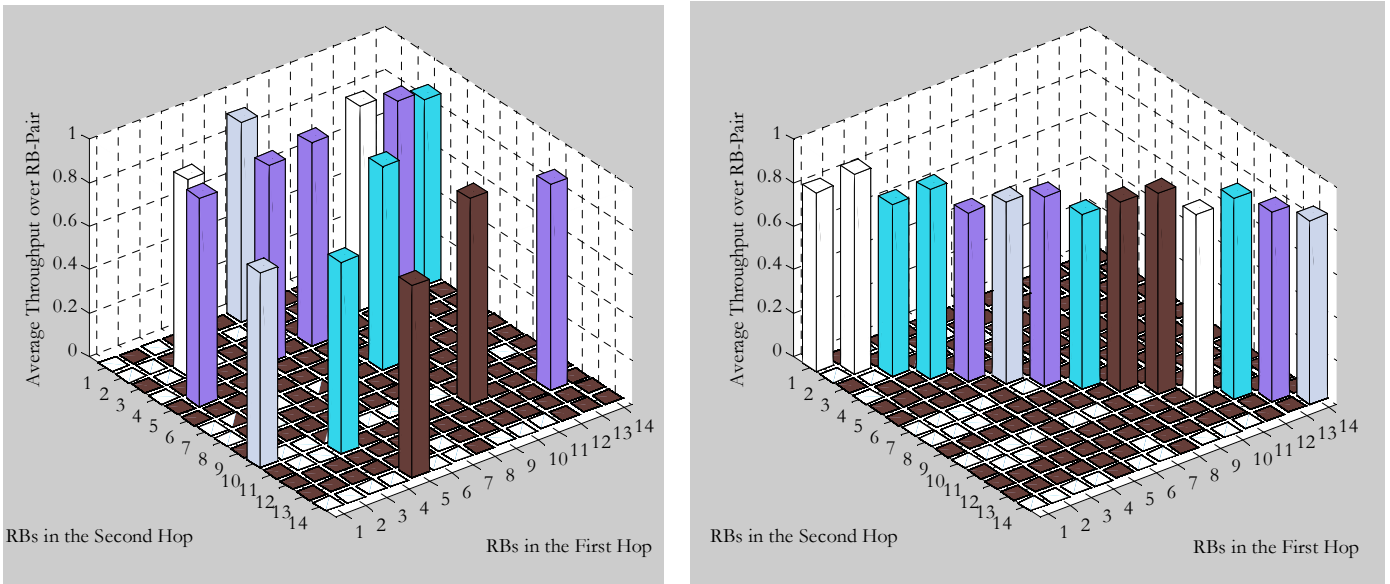
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(a) (b)
Figure 8. Snapshot of Simulated RB-Pairing for AF (a) SRBP (b) FRBP



(a) (b)
Figure 9. Snapshot of Simulated RB-Pairing for DF (a) SRBP (b) FRBP