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Research Article

Model Predictive Control for Nonlinear Inverted Pendulum Mobile Robot with Disturbance

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ABSTRACT

The Model Predictive Control is a control technique that has been greatly investigated in recent years. In this paper, nonlinear model predictive control is applied to an inverted pendulum-cart dynamic system we know that the inverted pendulum-cart dynamic system is an inherently unstable and nonlinear system.The controller is robust to parameter uncertainty and disturbances so that it is suitable for controlling an inverted pendulum system. Based on testing with step and sine reference signals without interference, the controller can stabilize the system well and has a fast response. Here the controller aim is to move the cart to a desired position and balancing the pendulum in upright position. the Model Predictive controller is robust against these changes and able to make the system reach the reference value.

The simulation results show that the proposed controller achieves an excellent performance. Here the modeling and simulation of the controller are carried out using MATLAB-SIMULINK and simulation results

Keywords:— Model predictive control, Inverted pendulum cart system, nonlinear system–dynamics, modeling

INTRODUCTION

Say that the pendulum will simply fall if the cart is not moved to balance it. Furthermore, the dynamics of the system is nonlinear. The objective of the control system is to balance the inverted pendulum by applying force to the carriage to which the pendulum is attached. A concrete example that relates directly to this inverted pendulum system is the attitude control of a booster rocket during takeoff.

The inverted pendulum problem was selected because it displays nonlinear dynamic behaviour, it is unstable about the desired operating point (pendulum standing up), and it is non-minimum phase. As an aside, it is representative of some practical applications. The Segway PT is a two wheeled (in parallel), self-balancing vehicle that transports a single person which uses the properties of the inverted pendulum. A walking humanoid robot displays inverted pendulum characteristics [16]. In literature, different versions of the inverted pendulum systems are presented. The familiar versions of the inverted pendulum are the cart inverted pendulum [1], the double inverted pendulum [2], the rotational single arm pendulum [3], the rotational two-link pendulum [4] and the triple inverted pendulum [5]. Since the last 7 decade, the cart inverted pendulum is widely used as a benchmark control problem [6] which has three different control tasks. The first one is a tracking control of the pendulum [7]. The second one is the swing up of the pendulum from the stable equilibrium point to the unstable equilibrium point [8]. The third one is the stabilization of the inverted pendulum around the unstable equilibrium point and driving the cart to the desired position [9].

LITERATURE REVIEW

Many modern and classical control strategies may be designed, tested, evaluated, and compared in a highly unstable inverted pendulum system. The control challenge involves creating dynamic models of systems for achieving the required response and performance of the system, therefore control law and dynamic model are employed. Non-linear control systems such as robotic systems, missile systems and many other non-linear control systems have been controlled using linear control methods such as PID control [6], [7].

Some variations of nonlinear controllers that can overcome parameter uncertainty and disturbance are adaptive modele predictive control [44]–[47]. The adaptive modele predictive control is good to solve parameter uncertainty and disturbance [48]. Meanwhile, controller gives a good system response. Therefore, by combining those nonlinear controllers, the proposed controller can solve parameter uncertainty and disturbance with a good response.

Model Predictive Control (MPC) was used in this research, mainly for its abilities to adapt to changes in the system. MPC is known for its accuracy, due to the prediction algorithm incorporated, along with the capability of disturbance rejection. Advanced control algorithms have been implemented in powertrains for some time now; the use of MPC pushes this boundary further, and will be important in the inverted pendulum cart in the near future, due to the ability to adjust the control law mid-execution. In this research, a basic explanation of MPC is presented to highlight the mathematics behind the controller. In the later sections, MPC will be applied to various powertrain configurations.[17]

MPC is a predictive control law in which the control input of the current step is calculated using a prediction of model behavior over a set future horizon. This type of control reflects human behavior and how we make decisions based on what we think the outcome will be. MPC tries to emulate this human thinking in anticipating changes and adjusting the control accordingly. The idea of predicting a set distance into the future is called the receding horizon concept [18].

The algorithm is designed to continuously update information and replace past steps. For each iteration, the model looks one step further in the future, keeping the horizon at its fixed and predetermined length. As with any predictive control law, MPC must have an accurate model of the system, in order to determine future system behavior. One of the key aspects of MPC is the ability to reject disturbances and correct for uncertainties; thus, the prediction model does not need to be perfect. Another useful component of MPC is that the models can be linear or nonlinear, which is not the case for many controllers [9].

Once the model predictions are calculated, the resulting error between the reference and prediction is minimized using a cost function and constraints. From here, the control input is determined. The process repeats itself each step. A block diagram representation of an MPC system is shown below:

This paper has five sections. Section one gives a brief idea about the paper. Section two deals with the nonlinear modeling of the system. Section three describes the designing of various control strategies. Section four depicts the simulation results and observations. The paper concludes with the results obtained in section five.

METHODOLOGY

2 Description mathematical modelling

2.1 Inverted pendulum system equations

Here we consider a pendulum cart system Figure 2-1 represents the free body diagram of the system. Here we assume that the rod of the pendulum is mass. Less and the hinge to which the pendulum is fixed is friction less. The mass of pendulum is concentrated at the center of gravity of the pendulum which is located at pendulum ball's center, the mass of the cart is represented as Mc and the mass of pendulum is represented as mp. The control force F acts along the x direction of the cart.

The rod's length is represented as l. The angle by which the pendulum is tilted represented as [4]

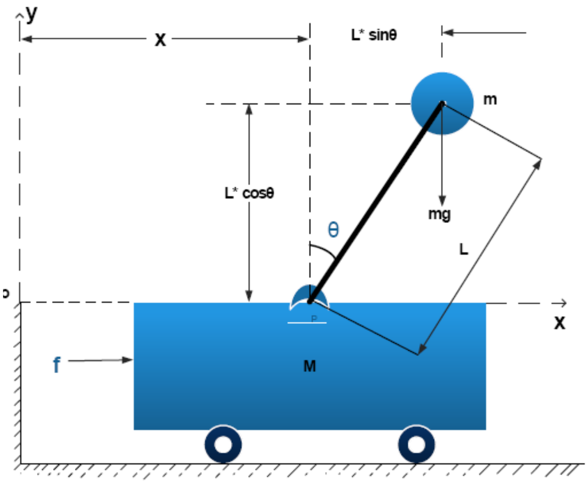


Fig1: Modeling of an Inverted Pendulum on Cart

A. Equation

The lagrange's equations of the inverted pendulum on a cart are given as follows

$$\frac{d}{dt}\left(\frac{\partial L}{\partial x}\right)-\frac{\partial L}{\partial x}=F$$
$$\frac{d}{dt}\left(\frac{\partial L}{\partial \theta}\right)-\frac{\partial L}{\partial \theta}=0 \tag{1}$$

Where $L = K - P$. By putting the expression of L in (1) and (2) and after solving it, lagrange's equations of the inverted pendulum on a cart can be expressed as

$$F=(m_p+m_c)\ddot{x}+lm_p cos\theta\ddot{\theta}-lm_p\dot{\theta}^2 sin\theta$$
$$0=lm_p cos\theta\ddot{x}+l^2m_p\ddot{\theta}-glm_p sin\theta \tag{2}$$

Where m_p, m_c, g and l are mass of the pendulum, mass of the cart, the gravity acceleration constant, and the length of the pendulum, respectively. The cart position and the rod angular displacement are denoted by x and θ, respectively. The physical system is shown in figure 1. The state space representation of the system is obtained by choosing the cart applied force F as the system input, the nonlinear model of an inverted pendulum system can be written as:

$$\dot{x}_1=x_2$$
$$x_2=f(x,t)+g(x,t)u+d(x,t) \tag{3}$$

The $f(x,t)$ is a nonlinear function of the system's states and $g(x,t)$ is a nonlinear function of the system's input. Meanwhile, $d(x,t)$ is the function of parameter uncertainty and system's disturbance.

And to overcome the under actuated problem, the output is set to be the mass position as

$$y=x+lsin\theta \tag{4}$$

The cart system is represented using nonlinear equations. The state space modelling is employed

$$\ddot{\theta}=\frac{Fcos\theta-(M+m)gsin\theta+ml\dot{\theta}^2cos\theta sin\theta}{mlcos^2\theta-(M+m)l}$$
$$\ddot{x}=\frac{F+ml\dot{\theta}^2sin\theta-mgsin\theta cos\theta}{(M+m)-mcos^2\theta} \tag{5}$$

The mathematical model of the system can be obtained by deriving associated lagrangian equations. Since the procedure well know. The details are skipped and the final model is referred to [x] for derivations. In the above, g

represents the graviationnal constant by defining the state vector as $x = [\theta, \dot{\theta}, x, \dot{x}]$, the state space model can be expressed as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = \frac{Fcosx_1 - (M+m)gsinx_1 + mlx_2^2 cosx_1 sinx_1}{mlcos^2 x_1 - (M+m)l}$$
$$\dot{x}_3 = x_4 \tag{6}$$
$$\dot{x}_4 = \frac{F + mlx_2^2 sinx_1 - mgsinx_1 cosx_1}{mlcos^2 x_1 - (M+m)l}$$

IMPLEMENTATION

3.MPC Inverted Pendulum System

MPC is a predictive control law in which the control input of the current step is calculated using a prediction of model behavior over a set future horizon. This type of control reflects human behavior and how we make decisions based on what we think the outcome will be. An example is deciding when to brake based on an obstacle in the road ahead of you, or any time when a decision has to be made before an event happens. MPC tries to emulate this human thinking in anticipating changes and adjusting the control accordingly. The idea of predicting a set distance into the future is called the receding horizon concept. The algorithm is designed to continuously update information and replace past steps. For each iteration, the model looks one step further in the future, keeping the horizon at its fixed and predetermined length. As with any predictive control law, MPC must have an accurate model of the system, in order to determine future system behavior. One of the key aspects of MPC is the ability to reject disturbances and correct for uncertainties; thus, the prediction model does not need to be perfect. Another useful component of MPC is that the models can be linear or nonlinear, which is not the case for many controllers. Once the model predictions are calculated, the resulting error between the reference and prediction is minimized using a cost function and constraints. From here, the control input is determined. The process repeats itself each step. A block diagram representation of an MPC system is shown below:

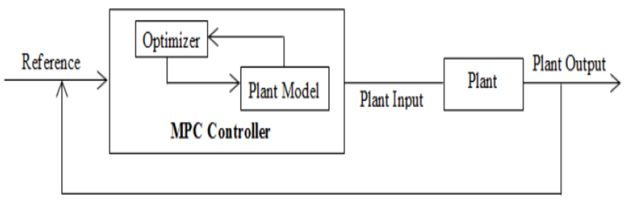


Fig2: Feedback of an Inverted Pendulum on Cart

3.1Model Predictive Controller Design Parameters

Designing the MPC controller takes into consideration the required constraints such as the steering angle limits. Figure 6 presents the main parameters and terms of the MPC controller, where the following nomenclature applies: *k* is the current sampling step and *Ts* the Control Time Step. Prediction horizon (P): number of time steps (the time on which the MPC controller looks forward to the future to make the prediction). Control Horizon (M): number of the possible control moves to time step *k+P*. The design parameters of the MPC controller are very important as this affects the performance and the computational complexity of solving the optimization problem. The choice of the design parameters should achieve the balance between the computational load and the performance. There are general recommendations, which can be taken into consideration for the parameters.

Sample time (*TS*): determines the rate that the controller executes the control algorithm. In the case of Control Time Step *Ts* interval is too long, the controller will not be able to respond in time to the disturbance, which means that the performance will be negatively affected. On the other hand, if *Ts* is too short, the controller's response will be faster, but this causes a significant increase in computational load. The recommendation, in this case, is to choose *Ts* between 10 to 20 samples of the Rise Time *Tr* in an open-loop system, where *Tr* is the required time that the response takes to rise from 10 % to 90% of the steady-state as Figure 7 shows [15].
Prediction horizon (P): should be chosen in a way that covers the dynamic changes of the system and the recommendation are to choose P to have 20 to 30 of samples covering the open-loop transit system response [15], [18], [26] and [29].
Control Horizon (M): Only the two control moves have a significant impact on the response behavior, choosing a large control horizon will only increase the computation complexity, based on that, the recommendation is to choose M to be 10 to 20 of the prediction horizon. A small value of M provides stability while in contrast, large

values reduce the robustness. It is recommended to choose M to be between 3-5 – as presented in [9], [15], [18], and [25].
For the model in this paper, the following strategy was used in order to choose the parameters which achieve satisfactory control performance: First, we initialized the parameters based on the recommendations above regarding the Sample Time, Prediction Horizon, and Control Horizon. Next step, is about tuning the parameters and then evaluating the MPC controller performance using the MPC Designer MATLAB toolbox until the optimal values provided the best control performance were determined. The weights of the inputs and outputs were determined using the MPC Designer by setting nonzero values to the inputs and outputs which need to track a reference value. Based on that, the weight equal is set to zero for the steering angle as it does not track a target. The weight of the Lateral Position and Yaw angle were determined with nonzero values as the main objective is position tracing.

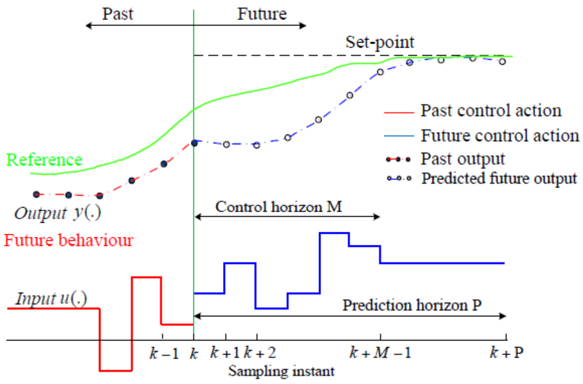


Fig3: Principle of the receding horizon

Consider a discrete time constrained linear time invariant system [18] , [9]

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \\ x_{min} &\leq x(k) \leq x_{max} \\ u_{min} &\leq u(k) \leq u_{max} \end{aligned} \tag{7}$$

Assume that pairs (A, C) and (A, B) are observable and controllable, respectively. Where A, B and C can be calculated from [8]. x_{min} , x_{max} , u_{min}, u_{max} are the minimum and maximum state and input constraints, respectively. The MPC solution that ensures the stability of the constrained problem in [8] can be obtained by solving the following optimisation problem at each sample:

$$J(u, x(k)) = x^T(k+N)\overline{Q}x(k+N) + \sum_{i=0}^{N-1} x^T(k+i)\,Qx(k+i) + u^T(k+i)Rx(k+i) \tag{8}$$

$$u_{min} = \begin{bmatrix} u(k) \\ \vdots \\ u(k+N-1) \end{bmatrix}^T \tag{9}$$

$$\begin{aligned} x_{min} &\leq x(k+i) \leq x_{max} \\ u_{min} &\leq u(k+i) \leq u_{max} \end{aligned} \tag{10}$$

$$\begin{cases} x(k+i-1) = Ax(k+i) + Bu(k+i) & i \geq 0 \\ y(k+i-1) = \ Cx(k+i) \end{cases} \tag{11}$$

Where $x(k+i)$ is the vector of predicted states at instant k.
N is the prediction horizon $Q \geq 0$, and $R > 0$ are a tuning parameter. $\overline{Q} > 0$ is the penalty on the final states. At each sample. The optimal sequence
$U = [u^*(k) \quad u^*(k+1) \dots \dots . . u^*(k+N-1)]^T$ is obtained by solving (9). The first row of the optimal sequence is applied to the system according to the receding horizon policy. The predicted states can be written in matrix form as:

$$X = Mx(k) + PU$$

$$\text{Where } M = \begin{bmatrix} A \\ A^2 \\ \vdots \\ A^N \end{bmatrix}, \quad P = \begin{bmatrix} B & 0 & \cdots & 0 \\ AB & B & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ A^{N-2}B & \vdots & \vdots & 0 \\ A^{N-1}B & \cdots & AB & B \end{bmatrix} \tag{12}$$

$$X = \begin{bmatrix} x(k+1) \\ x(k+2) \\ \vdots \\ x(k+N) \end{bmatrix}, \quad U = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+N-1) \end{bmatrix} \tag{13}$$

Using equation (13), the optimisation problem in (9) can be rewritten as:

$$J(x(k)) = \frac{1}{2}x^T(k)Yx(k) + u_{min}\left\{\frac{1}{2}U^T HU + x^T(k)Fx(k)\right\} \tag{14}$$
$$GU \leq W_e + Ex(k)$$

Where G, W_e, E are constant matrices that can be constructed from the constraints in (8).

$$H = 2(P^T\tilde{Q}P + \tilde{R}), \quad F = P^T\tilde{Q}M$$
$$Y = 2(M^T\tilde{Q}M + Q). \tag{15}$$

Where $\tilde{Q}$and $\tilde{R}$ can be constructed as follows:

$$\tilde{Q} = \begin{bmatrix} Q & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & Q & 0 \\ 0 & \cdots & 0 & \overline{Q} \end{bmatrix} \quad \overline{R} = \begin{bmatrix} R & 0 & \cdots & 0 \\ 0 & \cdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & R & 0 \\ 0 & \cdots & 0 & R \end{bmatrix} \tag{16}$$

Note that, the matrices G, W_e, E, H, F and Y can be computed offline

RESULTS AND DISCUSSION

The simulation results using the MATLAB-SIMULINK software package and the motor parameters listed in Table1. Internal and measurement zero-mean white Gaussian noises are injected.

Table 1. The parameters of the inverted pendulum on a cart

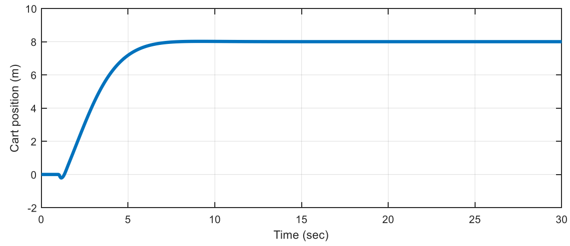
Symbol	Parameter	Value
M	Mass of cart	1
m	Mass of pendulum	1
L	The length of rod	0,5
g	Gravity	9,81
k	friction coefficient of cart	10

Assume the following initial conditions for the cart/pendulum assembly:
The cart is stationary at $\boldsymbol{x = 0}$
The inverted pendulum is stationary at the upright position $\boldsymbol{\theta = 0}$
The control objectives are:
Cart can be moved to a new position between -8 and 8 with a set point change.

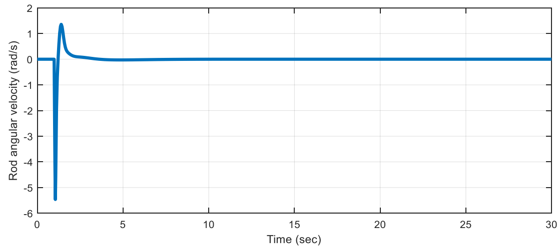
When tracking such a set point change, the rise time should be less than 4 seconds (for performance) and theovershoot should be less than 5 percent (for robustness).
The final scenario is the tacking of a desired cart position in front of the applied disturbance.
Validate the MPC design with a closed-loop simulation in Simulink.
The nonlinear simulation, all the control objectives are successfully achieved.

4.1Tracking Test

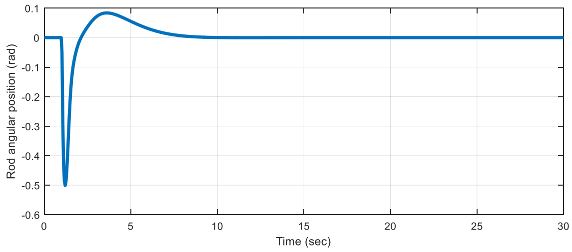
In this test, the tracking of the cart position to a desired position as well as the regulation of the rod angular displacement are considered. Assume that, the rod angular displacement is set at 20 degree and the desired position is 8 meter. Figure 4 shows that the proposed controller is able to derive the cart to the required position and the rod angular displacement is derived to the equilibrium point. The control action is shown in Figure 5.



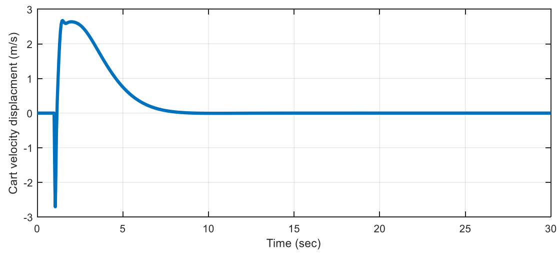
(a)



(b)



(c)



(d)

*Fig4:*Tracking test with disturbance (a) cart position (b) cart velocity (c) rod angular displacement (d) rod angular velocity

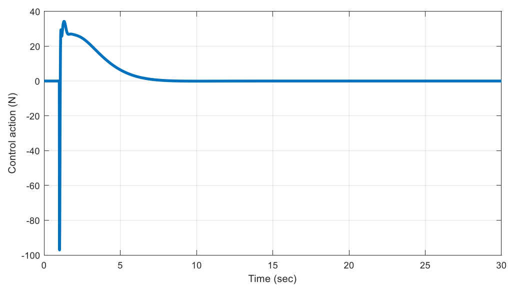
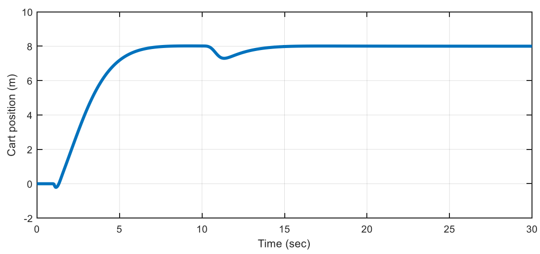


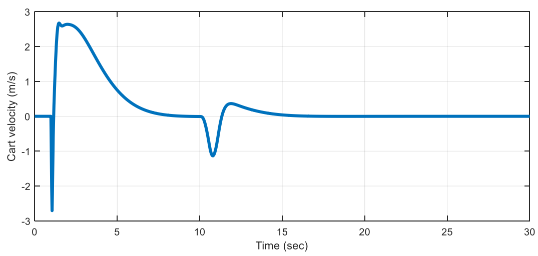
Fig5: Control action for the regulation test

4.2 Tracking with Applied Disturbance Test

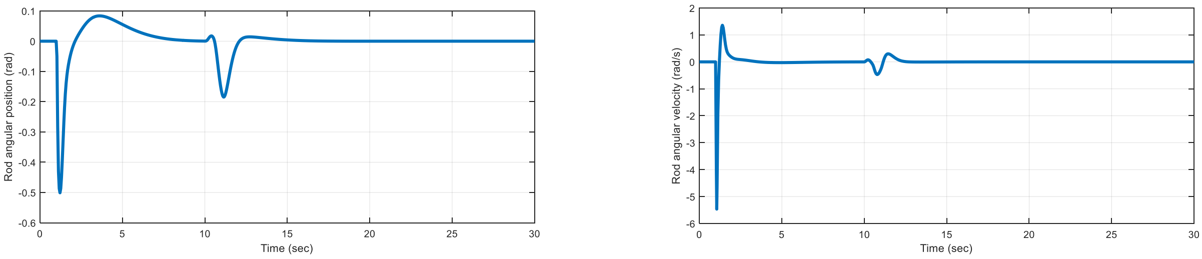
In this test, a step change in the rod angular displacement is applied to the previous tracking test. The objective is to test the robustness of the proposed controller. Figure 6 illustrates the step change in the rod angular displacement. The controller performance is shown in Figure 7. As shown in Figure 7, the proposed controller has the ability to reject the system disturbance. The control action is shown in Figure 7.



(a)



(b)



(c) *(d)*

Fig 6: Tracking test with disturbance (a) cart position (b) cart velocity (c) rod angular displacement (d) rod angular velocity

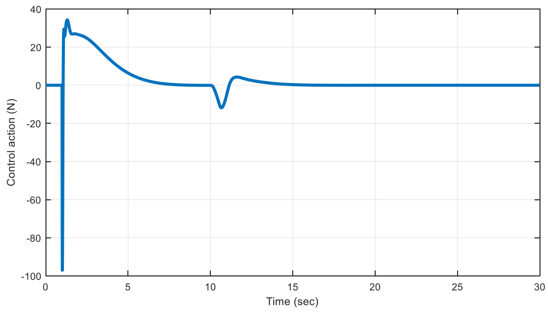
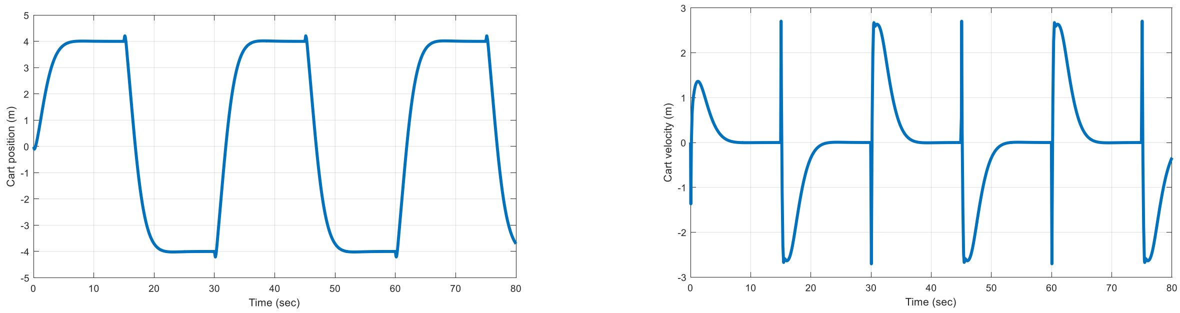


Fig 7: Control action for the regulation test

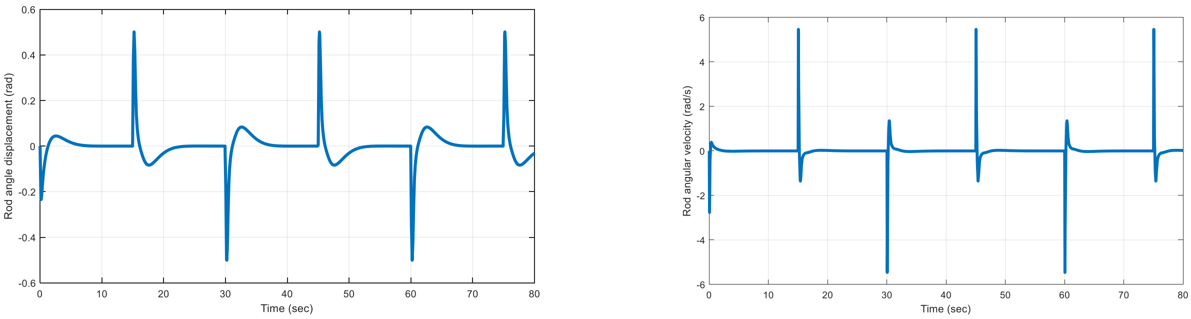
4.3Tracking Test in the two directions

The control objectives are:

Cart can be moved on the right and left with a new cart position between -4 and 4 with a s change.



(a) *(b)*



(c) *(d)*

Fig 8: Rod Regulation to the equilibrium point (a) cart position (b) cart velocity (c) rod angular displacement (d) rod angular velocity

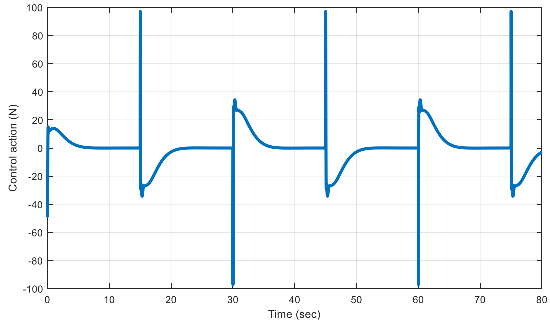


Fig 9: Control action for the regulation test

4.4 Tracking with Applied Disturbance Test

The objective is to test the robustness of the proposed controller. The desired position is -4 and 4 meter. Figure 5 shows that the proposed controller is able to derive the cart to the required position and the rod angular displacement is derived to the equilibrium point. The control action is shown in Figure 11.

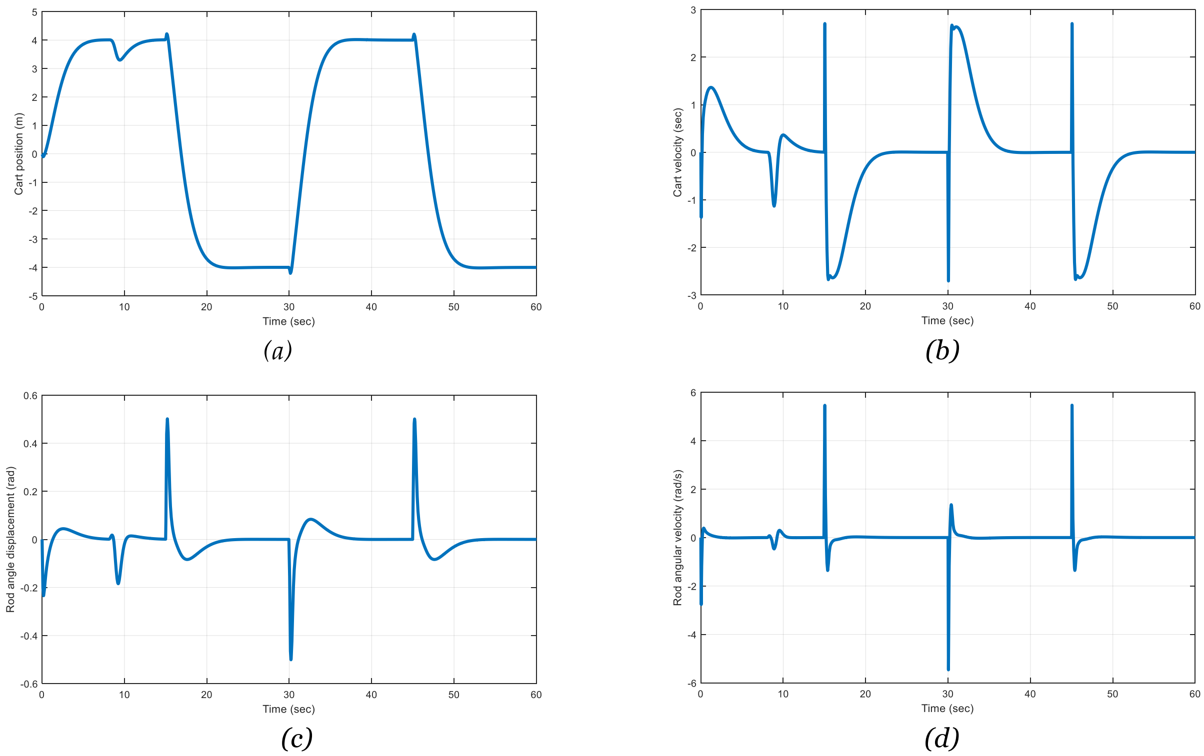


Fig 10: Tracking test with disturbance (a) cart position (b) cart velocity (c) rod angular displacement (d) rod angular velocity

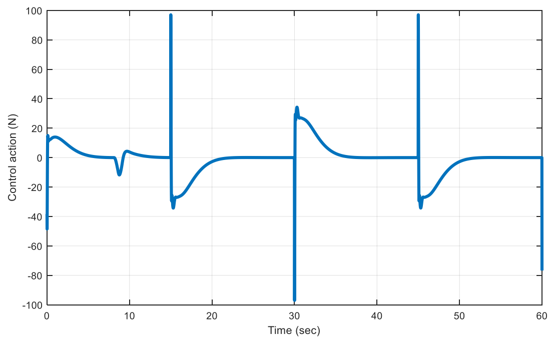


Fig 11: Control action for the regulation test

CONCLUSION

This paper presented an evaluation of a modern controller MPC to control the nonlinear inverted pendulum cart system. It is analysed for cases of using disturbance and without disturbance is executed. The pendulum stabilies in the vertical position and the cart reaches the desired position even in the presence of disturbance input. The performance of MPC method is superior than PID and LQR method, effective, robust and simple. The simulation results shows that the proposed controller has to the ability to regulate the rod angular displacement, derive the cart to the desired position and reject the system disturbance. The control design of MPC guaranteed the system converge in a finite time, there for proposed paper shall be helpful for researchers in the field of nonlinear applications such as control of aircraft, missile and satellites

ETHICAL DECLARATION

Conflict of interest: No declaration required. **Financing:** No reporting required. **Peer review:** Double anonymous peer review.

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