Modeling Poisson Error Process on Wireless Channels

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Abstract: Burst error modeling has seen extensive research and progress over several decades evolving into ever more complex modeling techniques used today. This paper analyzed usefulness of some prominent generative and descriptive (analytical) methods. Data containing error bits and packets from real wireless transmission was captured on a physical interface and used to obtain statistical information about error burst and gap behavior in the channel. Generative and descriptive modeling techniques were then applied to model the error process with the goal of establishing advantages and disadvantages of each technique. Generative methods were represented by the commonly implemented Elliot's model with parameters calculated using a generalized algebraic form. Descriptive methods were represented by 2 models; one of the most flexible exponentially shaped distributions with regard to parameterization and heavy-tailed function modeling - gamma distribution model, and a model utilizing a novel parameterization approach for the Markov modulated Poisson process (MMPP-2), producing second-order hyper-exponentially distributed characteristics. Results of the experiments were highly in favor of MMPP-2 model using a novel parameterization approach, demonstrating capability of MMPP-2 to model heavily interfered wireless channels exhibiting exponentially-shaped error.

Keywords: generator, Elliot's model, MMPP-2, gamma distribution, bit error, wireless channel.

1. Introduction

The popularity and availability of wireless technology inspired extensive research in areas associated with wireless systems. Especially in its initial stages of research and development it is preferable to test the concepts and their realizations in a controlled and simulated environment using mathematical models, rather than build an entire wireless system itself. Mathematical models have to be precise enough to disqualify inefficient or unrealizable concepts, but mathematically tractable over a reasonable simulation time period.

There are 2 conceptual approaches to wireless channel modeling: modeling of the propagation channel's physical characteristics and modeling of statistical characteristics of the underlying channel error process. An example of the physical propagation channel model application is demonstrated on a mix shadowed Rician and Nakagami channels in [1]. Knowledge of error process and its statistical characteristics is beneficial for optimization of wireless transmission systems on protocol and error control level, as demonstrated e.g. in [2]. In order to observe the nature of errors in the channel, a trace must be captured first by mathematically relating the output data sequence at the transmitter with the data sequence received by the receiver. The resulting trace consists of zeros and ones representing correctly received bits and error bits respectively. Consecutive error bits are referred to as error burst, whilst consecutive correctly received bits are referred to as error gaps or gaps.

2. Related work

The first widely accepted error model survey was published by Kanal and Sastry [3] in 1978 and it classifies error models as either generative or descriptive. Generative techniques are those that use underlying mechanism to describe the channel (e.g. Markov chains) and descriptive techniques aim to fit specific stochastic properties of the observed trace with stochastic distributions (e.g. Pareto and Gamma distribution model).

Because of their wide-scale application and easy realization generative models based on Markov chains are extremely popular in error modeling even nowadays. The most widely applied generative models are Gilbert's [4], Elliot's [5] and Fritchmann's along with their many modifications.

Descriptive models typically use stochastic distributions and model several moments of the communication link error process. Estimation of distribution parameters precedes modeling and the most commonly used probability density functions can be moved, stretched, shaped, altered, or any combination of these features, using up to 3 parameters. Phase-type distributions hold special position regarding the total number of parameters; they are defined by a multiplication of mixture's base stochastic distribution's number of parameters and the total number mixture components. Both generative and descriptive modeling approaches offer different advantages and disadvantages and are with varying success used for different purposes..

Later progress in error modeling introduced new mathematical concepts and model classes, often referred to as pure models: semi-Markov models, Hidden Markov Models (HMMs), empirical approaches including algorithmic models, chaos models, Deterministic Process Based Generative Models (DPBGM) and Stochastic Context-Free Grammar models (SCFG).

More recent trend is to combine individual pure models into new configurations known as extended models, which exploit advantages of component pure models to create an error model with either fewer issues or more beneficial properties (e.g. [6] and [7]).

The most current approaches aim to design models that could be parameterized adaptively in a bit-by-bit fashion and be able to capture faster rate of change [8].

Error process observations of real data confirm that an independent channel is not a feasible solution for many applications [3]. A generalized Markov model construction for partially dependent events in a form of cascade Gilbert model is presented in [6] and later extended to a cascaded combination of Gilbert and Elliot models in [7].

Based on the real data, the goal of this study was to build on results and knowledge obtained from [6] and [7] and to use the Elliot's model as a reference to compare with empirical (descriptive) models, one of which is parameterized using a novel adaptive parameterization method. In case one of the proposed descriptive models wes sufficiently precise, it would be possible to modify one stage of cascade model from [7] and improve the model's overall efficiency. Wireless transmission is most sensitive to small-scale error process typically in a bit scale, therefore the error sequences and error gap process was modeled in this work, omitting the packet model presented in both [6] and [7]. The effects of large bursts causing packet damage and subsequent loss usually dissipate much faster and subsequent multiple packet loss caused by a single burst is comparatively lower than in case of short bursts occurring on the bit level of a single packet. Moreover the observations of processes on real data confirm that large-scale and small-scale bursts occur relatively independently and therefore it is possible to model them separately. The channel is assumed to be stationary over the observed period.

3. Theoretical basis and model

Simulation of wireless networks requires a statistically or deterministically precise channel model describing the wireless link characteristics. Knowledge of particular channel's error process is imperative for adjusting the error control schemes to a particular network or a specific situation within the network. Stochastic error process description requires ex ante knowledge of the channel's statistical properties, especially the statistical properties of error sequences and error gaps produced by the process during an error burst.

An error process on a digital communication link can be considered a binary discrete-time stochastic process. If I is a countable set of integers $t \in I$, a_t the digital input sequence, b_t the corresponding output sequence and n_t the noise sequence representing the effect of the channel on the data, also referred to as *trace*, then:

$$b_t = a_t + n_t \qquad (1)$$

A correctly received bit is represented by "0", incorrectly received bit is represented by "1'. Extraction of error source features can be performed bitwise and error modeling then becomes equivalent to statistically correct modeling of the trace. Consecutive sequence of "1" is called an error burst. A gap may be defined as a sequence of consecutive "0" between two "1" and represents the distance in bits of two neighboring bursts. Empirically the shortest error gap (expressed by (2) taken from [7]) or error burst has length 1 [3]. The error overflow assumption that the last "1" of the

previous packet and the first "1" in the following error packet are not part of the same burst error and containing the error burst within the packet limits in [7] were applied as well.

$$\overline{G}(n) = \sum_{j=0}^{\infty} (j+1) \frac{\text{gaps in } (j+1) \text{ long n-bit packet}}{\text{all gaps in n-bit packet}} \qquad (2)$$

The trace used in this study is identical with the data collected and used in [7] where a thorough description of data set capturing procedure is documented. Following statistics about the channel behavior were extracted during the analysis phase:

- Small-scale (intra-packet) error burst length distribution
- Small-scale (intra-packet) error gap length distribution
- Total bit error probability of clusters with defined size

3.1 Applied models

Models proposed for analysis are further described along with the methods for obtaining the parameters of these models. The following section then contains estimated parameters used in simulations to obtain the results.

3.1.1 Generalized Elliot's model

Generalized Elliot's model [9] is based on Elliot's original work [5] on Markov chain bit error generator (fig.1) and extends it by using an algebraic form with transitional and generating matrix to an arbitrary number of states, contrary to Elliot's original model proposal of 2 states.
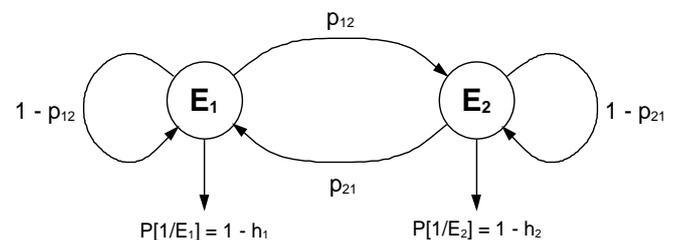

Figure 1. (Generalized) Elliot's bit error model

Common notation with the work of Siran and Maly [9] is used throughout this article to define key variables of the Elliot's model.

Final probability state vector of the model is:

$$\boldsymbol{\pi} = (\pi_1, \pi_2) \qquad (3)$$

Generator matrix for the process modeled by Elliot's model:

$$\boldsymbol{H} = \begin{pmatrix} h_1 & 0 \\ 0 & h_2 \end{pmatrix} \qquad (4)$$

Transition probability matrix with transitional probability in fig. 1:

$$\boldsymbol{P} = \begin{pmatrix} 1 - p_{12} & p_{12} \\ p_{21} & 1 - p_{21} \end{pmatrix} \qquad (5)$$

Then according to [3] the final probability state vector can be rewritten as:

$$\boldsymbol{\pi} = \left(\frac{p_{21}}{p_{12} + p_{21}}, \frac{p_{12}}{p_{12} + p_{21}} \right) \qquad (6)$$

Central to generalized model's parameter calculation is the nonlinear equation (7): probability that n units long cluster of correctly received bits is generated from the generalized

Elliot's model is:

$$p(n) = \pi (PH)^n \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad (7)$$

Parameters necessary for proper modeling of a wireless channel are for a 2 state Elliot's model limited to knowledge of p_{12}, p_{21}, h_1 and h_2 that can be established by solving a series of nonlinear equations (7).

3.1.2   Gamma distribution model

Generalized Gamma distribution is typically used to describe variables bound on one side. A stochastic process with a mean and a variance can be approximated by the gamma distribution function using it's 3 parameters – location (a), scale (b) and shape (c).

Gamma distribution can be favorably used to model stochastic processes with precision up to and including the second moment.

Its probability density function is:

$$f(x) = \begin{cases} \dfrac{1}{\Gamma(c)} b^{-c} (x-a)^{c-1} e^{-\frac{x-a}{b}} & x > a \\ \\ 0 & otherwise \end{cases} \qquad (8)$$

Where:

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \qquad (9)$$

The generalized gamma distribution's moments are easily expressed using its parameters by linear (1st moment) and quadratic (2nd moment) function, which is not the case with other distributions that can be derived from gamma distribution – e.g. Weibull, Rayleigh and other. Mean and variance of the gamma distribution:

$$\mu = a + bc \qquad (10)$$

$$\mathrm{var} = \sigma^2 = b^2 c \qquad (11)$$

Having observed the mean and variance from the trace it is easy to express (11) using (10) by omitting a as an unnecessary parameter for our purposes (since it can be included as shifting the entire generated set by adding the value a) to find parameter b and then go back to (10) to calculate the parameter c using an already known value of b. Modeling the error burst and error gap process then becomes a matter of observing the first and second moments of both processes in the trace and applying them equations (10) and (11).

3.1.3   Hyper-exponential distribution model

Hyper-exponential distribution describes a stochastic process that can be decomposed to a finite sum of exponential processes and used to emulate atypical exponentially shaped heavy-tailed distributions.

Probability density function of a hyper-exponential distribution for k components is:

$$f(x) = \sum_{i=1}^k p_i \lambda_i e^{-\lambda_i x} \qquad (12)$$

The process of parameter estimation for hyper-exponential distributions however is quite complex. Therefore many approaches to parameter calculation have been devised. Hyper-exponential parameter estimation [10] implemented in the initial stages of experimenting with this distribution produced an excellent cumulative density function for both error and gap processes, but the found parameters were unsuitable for a generating process, which failed to approximate moments of the stochastic processes.

An approach using Markov modulated Poisson process (MMPP-2) was therefore chosen instead. It also produces a hyper-exponentially distributed variable, however the parameter estimation process is different from the one applied in [10] and allows moment fitting of the observed random variable.
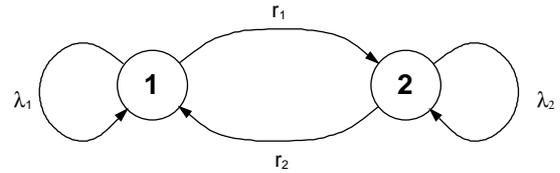

Figure 2. MMPP-2 model with parameters

The simplest form of a MMPP is the MMPP-2 model (fig. 2), where two independent Poisson processes with different arrival rate parameters λ_1 and λ_2 transition from one to the other at rate r_1 and r_2. The results of MMPP-2 traffic model versatility analysis for applications in ATM cell loss rate modeling [11] demonstrate the feasibility of MMPP-2 model for burst process modeling.

Assuming interval-stationary MMPP-2 processes, the inter-arrival time X_i between consecutive occurrences is a second order hyper-exponential distribution (H2) defined by the probability density function (13):

$$f(x) = q u_1^{-u_1 x} + (1-q) u_2^{-u_2 x} \qquad (13)$$

As demonstrated and derived in [12], the parameters of the hyper-exponential distribution function can be obtained from the MMPP-2 (fig. 2) using following substitutions:

$$u_1 = \frac{\lambda_1 + \lambda_2 + r_1 + r_2 - \delta}{2} \qquad (14)$$

$$u_2 = \frac{\lambda_1 + \lambda_2 + r_1 + r_2 + \delta}{2} \qquad (15)$$

$$q = \frac{\lambda_2 r_1^2 + \lambda_1^2 r_2 - \delta}{(\lambda_2 r_1 + \lambda_1 r_2)(u_1 - u_2)} - \frac{u_2}{u_1 - u_2} \qquad (16)$$

Where:

$$\delta = \sqrt{(\lambda_1 - \lambda_2 + r_1 - r_2)^2 + 4 r_1 r_2} \qquad (17)$$

Many approaches to estimating MMPP parameters λ_1, λ_2,

r_1 and r_2 have been proposed, but cell counting algorithm and moment fitting coupled with fitting of the auto-covariance algorithm presented in [13] belong to the most commonly referenced. Other algorithms for parameter extraction include nonlinear optimization [11] or histogram method [12]. Generally, all methods can be divided into either cell counting statistical methods or methods based on inter-arrival statistics.

Because the trace was available for statistical analysis, a novel approach eliminating posterior calculations necessary for parameter extraction was proposed for purposes of this research; all 4 MMPP-2 parameters were obtained directly from the trace using an adaptive approach.

Novel Parameterization algorithm

Consider a trace (fig. 3) that can be decomposed into consecutive error bursts and error gaps in the order as they appear in the trace. Because the gaps and bursts have different stochastic distributions, they are also modeled using 2 separate MMPP-2 submodels.
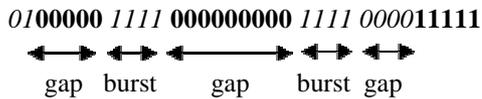
*0*1**00000** *1111* **000000000** *1111* **000**11111

gap burst   gap   burst gap

Figure 3. Example sequence extracted from the trace

The first step of the proposed algorithm is to choose a threshold value. Each MMPP-2 transitions between two Poisson processes generating either shorter or longer sequence and the threshold determines which one should be assigned generation of the currently processed burst or gap length. For typical applications the mean value will suffice as a viable threshold, for specialized applications the threshold can be estimated as a mode of the observed set or even be arbitrarily chosen. For the example trace the threshold for both gaps and bursts was set to 4 dividing the burst and gap lengths into 2 groups: smaller than or equal to (italic) and longer (bold) than the threshold. Parameters λ_1 and λ_2 representing the exponential distribution parameter can be obtained as an inverse value of the mean of all burst/gap lengths smaller than or equal to the threshold in the first case and longer than the threshold in the second case (the inverse value is in fact the effect of exponential distribution contribution to the Poisson process, the mean value of an exponential distribution is inverse to its only parameter, so must be the mean calculated from the example set). Transitions between these Poisson processes are then given by cumulative lengths τ_i, where τ_i represents the average distance (in bits) of 2 neighboring elements generated by the i-th Poisson process for the burst/gap above or below the threshold. Transition rates r_1 and r_2 can be obtained as an inverse value of the mean of corresponding τ_i. Having an example sequence (fig. 3), the parameters for the MMPP-2 model of the gap distribution are:

- $\lambda_1 = 1/(1/2*(1+4)) = 2/5$ - shorter gaps
- $\lambda_2 = 1/(1/2*(5+9)) = 1/7$ - longer gaps

- $r_1 = 1/(1/1*(23)) = 1/23$ - shorter gaps
- $r_2 = 1/(1/1*(4)) = 1/4$ - longer gaps

4. Results

Packets have been generated using all 3 models described in the previous section using Matlab (based on the concepts presented in section 3). PRNG is used to generate sequences of pseudorandom values from interval $<0,1>$ that are further transformed into the desired output values.

The first model is a pure generative Elliot's model whose output is a unique observable channel error process and its algorithmic representation is a simple if-else block scheme with parameters as threshold values; the first generated random value is used for state selection, the second is used for bit generation.

The second model is a combination of 2 descriptive gamma distributions modeling the small-scale channel error process, one the error bursts and the other error gaps, using the inverse transformation method.

The third constructed model descriptively models the small-scale intra-packet error burst and gaps, using a hyper-exponential distribution for each, utilizing a MMPP-2 model to estimate the parameters of the $H2$ distribution. Composite generation principle is used to obtain the burst/gap lengths using the obtained parameters.

There are several approaches to comparing these 3 models. One approach is to compare the models by bit error probability at different cluster lengths. Another approach would compare the generators from the histogram perspective of the method's ability to describe the error bursts and error gaps most realistically. Both types of bit error analysis of intra-packet small-scale error processes are necessary for establishing the quality of wireless channel error and gap process models for purposes of FEC design.

Following analysis of the results will be as thorough as it is necessary to qualitatively evaluate the models and found parameters will be presented to enable result verification.

4.1 Model parameters

Parameter expression methods for each technique used in this study were presented in section 3. Based on the results and conclusion located in [6] and [7], Elliot's model proved to be sufficiently precise in modeling the absolute and relative bit error probability, but could be improved in modeling gap and burst process. Descriptive approach was analyzed to answer the question whether error burst and gap process would not be more efficiently modeled using empirical modeling techniques.

Parameters for Elliot's model (tab. 1) are obtained by solving the system of nonlinear equations (7). The probability of at least one error bit in a sequence of length n is found by analyzing the trace for different values of n. Then MathCAD equation solver is used to find the solution of sought parameter values. The found solution does not have to be optimal, but causes the cluster probability to approximate the overall probability of at least one error bit in the entire cluster over the observed range (length of packet, set as 8480 bits).

Table 1. Elliot's model parameters

Parameter	Value
p_{12}	5.000000090198e-4
p_{21}	5.769910104226e-5
h_1	0.351021716978541
h_2	0.999885177348073

Gamma distribution parameters (tab. 2) are extracted using the observed mean and variance values of the error burst and error gap process in a received error packet from (10) and (11). The mean value of error gap and burst length along with the variance is computed from the available trace.

Table 2. Gamma model parameters

Parameter	Value
b_{gap}	5138.375364
c_{gap}	0.008202
b_{burst}	1556.487058
c_{burst}	0.001961

Hyper-exponential distribution parameters (tab. 3) and mixture components are estimated using the novel algorithm and equations outlined in section 3.1.3.

Table 3. Hyper-exponential model parameters

Parameter	Value
p_{gap_1}	0.9744
λ_{gap_1}	0.3983
p_{gap_2}	0.0256
λ_{gap_2}	0.0006
p_{burst_1}	0.7710
λ_{burst_1}	0.7323
p_{burst_3}	0.2290
λ_{burst_3}	0.1145

4.2 Cluster Error Analysis $P(1,n)$

Cluster is a grouping of several consequent bits from the observed sequence. If any of the bits in the cluster is equal to "1", meaning an error bit is present in the cluster, the entire cluster is considered as error cluster. Cluster analysis reflects the set's ability to hold multiple moments and internal set statistics. The relation to former definitions is $P(1,n) = 1 - p(n)$, where n is the cluster length. Cluster of length 1 in all cases represents the true bit probability, as only 1 bit is considered a cluster.

Results of the cluster analysis for all 3 models and the real data are depicted in fig. 4. The original data is represented by a filled black line.

Elliot's model generated data represented by the red (darker) dashed line have very similar cluster error probability as the real channel over the entire observed interval showing the reason for its wide application.

Green (or lighter) dashed line produced by the gamma distribution model competes with the MMPP-2 model, yet being visibly different from the real data, hence less optimal than the Elliot's model.
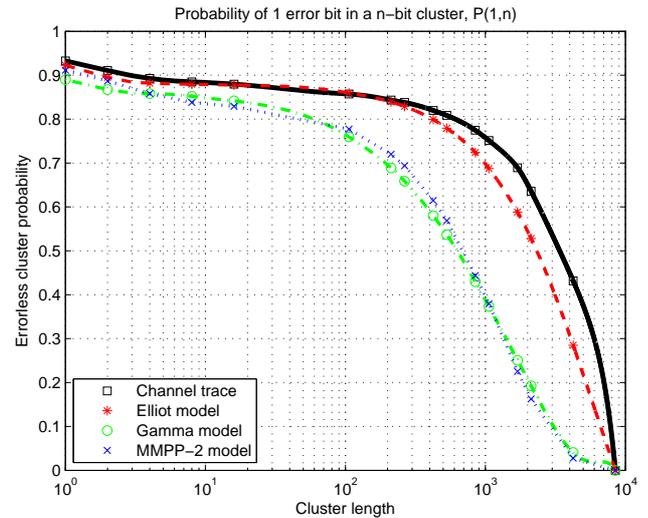

Figure 4. Probability of an errorless cluster in packets received with errors for real data, Elliot's model, gamma and hyper-exponentially generated data

4.3 Model parameters

Histograms presenting the results of the generating process for all models are depicted in two figures, both were showing the probability of occurrence of various burst/gap lengths. Figure 5 depicts the histogram of error bursts, while fig. 6 demonstrates the histogram of error gaps found in the error packets of the real and generated data sets.

Real channel data is depicted with the thick, black line to give reference. It is difficult to visually find a clearly superior approximation and therefore a mathematical approach using Jeffrey's divergence is used later on.
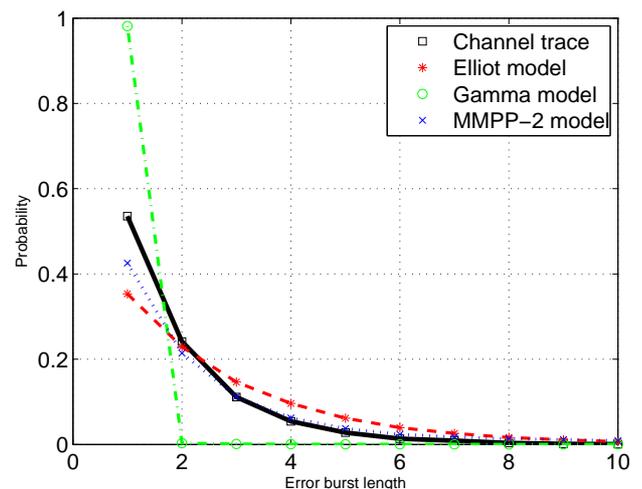

Figure 5. Burst error histogram for all models and the real channel data

The gamma distribution creates an anomaly (fig. 5) – a sharp peak – in case of error burst generation. This is an unexpected, yet explainable result. Considering that generating methods for gamma distribution mainly focus on generating random variable with shape parameter $c \geq 1$.

Generating a distribution with shape parameter c 1, such as the case of this modeling problem, is the domain of very specialized algorithms. One such recent algorithm [14] was used for experiments documented by this paper. The algorithm however wasn't able to generate error burst lengths and holding the desired gamma distribution shape. Surprisingly enough only 4x greater shape parameter, yet still too small for most generators (including the Matlab's implementation), was enough to model (fig. 6) the error gaps comparatively well as Elliot's model (visually).
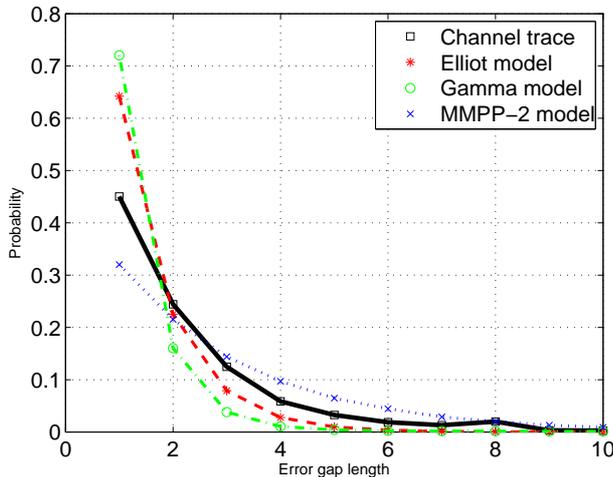

Figure 6. Error gap histogram for all models and the real channel data

Hyper-exponential distribution (MMPP-2 model) arguably has a visually more similar shape to the histogram obtained from the trace than both gamma and Elliot's models. Especially for error gaps the Elliot's model and gamma distribution are for gaps longer than 2 units visually less precise.

4.4 Jeffrey's divergence

Histograms obtained by analyzing error burst and gap process of the generated data were compared with the original histogram using the Jeffrey's divergence (18) taken from [15]. If H and K are two histograms, then Jeffrey's divergence is defined as:

$$ d_J(H,K) = \sum_i \left(h_i \log \frac{h_i}{m_i} + k_i \log \frac{k_i}{m_i} \right) \quad (18) $$

Where $m_i = (h_i + k_i)/2$. The smaller the distance, the more similarity exists between the two compared histograms on a bin-to-bin basis. Only the bins with identical value i are analyzed.

Results of the comparison of all models with the histogram produced by real trace using Jeffrey's divergence (tab. 4) are for both – burst and gap error modeling - in favor of the MMPP-2 model using a second-order hyper-exponential distribution, both visually from the histogram and numerically from the Jeffrey's divergence. Gamma distribution places third in the comparison both visually and numerically. The results confirm what can be empirically seen in fig. 5 and fig. 6.

Table 4. Jeffrey's divergence for generated histograms

	Burst Error histogram	**Gap Error Histogram**
Elliot	0.0690	0.0270
Gamma	0.1222	0.3566
MMPP-2	**0.0472**	**0.0202**

5. Conclusions

Data set extraction from physical layer of IEEE 802.11b and further modeling was performed at the Institute of Telecommunications, Slovak Technical University in Bratislava. Error burst and gap features of the captured traffic exhibited exponentially shaped heavy-tailed behavior in both processes' distributions, which is relatively common for wireless channels with heavy interference. Demonstrated modeling techniques verified applicability of gamma function and MMPP-2 modeling when compared to Elliot's model, used for instance in [7] to model the insta-packet small scale error process.

Gamma distribution seems to fail in modeling the basic properties of the channel due to lacking proper techniques for random variable generation of distributions with shape parameter c 1. Unless a more effective algorithm than [14] for generating the gamma random variable with extremely small shape parameters is proposed, modeling error burst and error gap process becomes a difficult and relatively ineffective, imprecise task.

On the other hand, the MMPP-2 could be further considered for bit-error burst and gap process modeling and could even be applied instead of the Elliot's model in a cascade model presented in [7]. A significant improvement to parameter extraction of MMPP-2 model in a form of novel proposed parameterization technique in section 3 makes it possible to estimate the hyper-exponential parameters dynamically and adaptively. The experimental results prove that it is a viable method for error process modeling and deserves more attention in the future. Also, threshold estimation remains a focal point of further research – proper threshold identification could improve the accuracy of the MMPP-2 model, albeit already more useful for error gap and burst modeling, than Elliot's model, regarding the probability histograms. In order to capture the cluster probability (fig.4) more efficiently, several new parameters could be added to produce a better $P(1,n)$ fit.

Generative and descriptive methods could be used together to improve the model characteristics, as demonstrated in this paper, modeling using MMPP-2 model can theoretically be even more precise than Elliot's model with the greatest advantage lying in the possibility to efficiently and easily obtaining the MMPP-2 parameters directly from the observed trace, whereas the Elliot's model requires significantly more calculations and solving of a set of nonlinear equations for parameter estimation.

Where the empirical methods fail to achieve the Elliot's model accuracy is cluster probability $1 - P(1,n)$ solely because the generalized Elliot's model is optimized to fit this statistic precisely.

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